P. HOLDEN

THE MONOTYPE RECORDER

VOLUME 40 WINTER 1956 NUMBER 4

Setting Mathematics

WITH A GLOSSARY OF MATHEMATICAL TERMS AND NOMENCLATURE OF SIGNS

by Arthur Phillips

TOGETHER WITH A NOTE ON THE SYSTEM OF 4-LINE MATHEMATICAL SETTING BY THE MONOTYPE CORPORATION LIMITED

THE MONOTYPE CORPORATION LIMITED

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A MATHEMATICAL SORTS LIST *which is complementary to this issue of the* RECORDER *will be sent free on application to*
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THE MONOTYPE RECORDER

VOLUME 40 NUMBER 4

SETTING MATHEMATICS

Editorial Note

THIS number of the MONOTYPE RECORDER has been written to help users of 'Monotype' machines who feel that they need more information or clarification on problems of mathematical setting. It is also hoped that many mathematical authors and editors will be interested in the subject-matter and that some of the examples will help them to understand the printer's problems.

The reader will naturally not expect to find any *official* pronouncement on Mathematical Style and Terminology from any institution, least of all from the Monotype Corporation, which must be all-things-to-all-printers throughout the learned world. But Mr. Phillips, though writing as an individual, has done so from an enviably central position.

We have asked him to extract from a longer monograph on which he is at work, such material as would in his experience best serve to answer the "general printer's" need for information.

The information on equipment has never before been made available, in so succinct a form, to the printer who finds that he must now include the facilities of mathematical setting in his service to the publisher. The choice of type size and face and its effect on the complexity of setting and equipment are discussed. Attention has been given to the problems of spacing and to the setting out of mathematical formulae which should assist the copy-preparer. Information on some of the mathematician's notation will help the printer to recognize the more obvious mathematical conventions.

The list of abbreviations provides an authoritative reference on the form of abbreviation, its meaning and the use of capitals. A glossary is included which will give the non-mathematical printer a brief glimpse of the technical vocabulary of the mathematician.

Examples have been numbered with decimal notation in the same manner as formulae in mathematical texts, although section numbers have been omitted.

Intentionally, several examples of displayed formulae have been badly set. These have been given bold reference numbers and an asterisk. Other examples are mathematically and typographically correct but are either not considered to be the best notation or have caused the printer more trouble than was necessary; these formulae also have bold numbers but no asterisk. It has not been possible to differentiate the few instances where correct and incorrect formulae bear the same number.

An attempt has been made to name all the mathematical signs that have been cut by the Monotype Corporation. It is obvious that this can be only partially successful, firstly because there is a continuous influx of new signs and secondly because signs are not always used for the same purpose. In more advanced mathematics it would be necessary to know the context in order to identify the exact meaning of the sign.

Mathematical Sorts List

A comprehensive list of mathematical sorts has now been prepared for the first time and is published separately; it can be obtained by completing and posting the enclosed postcard.

This is an essential reference book for any user of 'Monotype' machines who undertakes mathematical work. It includes alphabets of twenty-one different series which can be used in mathematical setting. The most important part of the sorts list is the matrix numbers of accented sorts. All Greek letters which bear the accents here shown on rho ($\dot{\rho}$ $\ddot{\rho}$ $\bar{\rho}$ $\bar{\bar{\rho}}$ $\tilde{\rho}$ $\hat{\rho}$ $\underset{.}{\rho}$) are listed alphabetically for Series 90, 91, 92, 106, 472, and 473. Italic capitals and lower-case letters are also shown with these same accents for Modern Series 1, Old Style Series 2, Modern Series 7, Imprint Series 101, Baskerville Series 169, Times New Roman Series 327. Other occasional accents with a mathematical significance are also included.

The matrix numbers of signs are given for series of matching signs which will enable the printer to ensure that matrices ordered are designed to work with the equipment that he already possesses.

The Four-line System

Readers of the MONOTYPE RECORDER who wish to obtain further information on the four-line setting of mathematics are invited to apply to The Monotype Corporation Ltd., attention of Service "A" Department, Salfords, Redhill, Surrey.

THE MONOTYPE RECORDER

VOLUME XL NUMBER FOUR

MATHEMATICAL NUMBER

Introduction: The Silent Language

As every printer knows, it takes more than 26 "soldiers of lead"—printing types of different "characters"—to "conquer the world" of today. Those of our citizens who are nearest to illiteracy seem quite content that their main reading-matter (the balloons in the comic strips) should be lettered in capitals only. But the soldiers increase in number as life becomes more complex. Midway on the scale are the catalogues and text-books which have to employ seven different alphabets. But in the rooms and classrooms where the mathematicians are testing and establishing the groundwork for all scientific thinking and invention which in turn will be the ground of all progress in modern civilization, people have to express values, quantities and relationships by symbols which differ basically from those of the alphabet, in that they have no fixed phonetic values. The primary thing about the alphabet, the thing that gave it its chance to conquer the western world, is its notion of making signs representing sounds instead of ideas. In the mathematician's world they still use sign language. How do you pronounce "2" or "3"? It depends whether you are speaking English or some other language. And even if you think of "6" as making the sound "six" you have only to move on to the symbols which express relationships and values to enter into a soundless world in which the most complex and delicate statements can be made, to anyone in the world who can read them without any trouble about the language barrier. Here writing for once wins hands down over oral speech. It is of course humanly possible to read out over the telephone the immediate text of some passage in an equation which the printer is being asked to correct; but the printer would do well to deprecate any such primitive and clumsy method of expressing abstract ideas.

And yet it is mattering more and more today to the general printers, that they should know enough about this "silent language" to be able to discuss with mathematicians and their publishers the problems of communication in this soundless world: say, such problems as the number of special signs and symbols available in the printer's cases or matrix-stores for this or that type of "formula", and what they consist of, and what others would have to be ordered. That last is a delicate point which sometimes has to be discussed in an oral interview, or over the telephone. Some select glossary of the most common or "accepted" mathematical terms and symbols is now essential to any but the smallest jobbing shops of the country.

The reason is, that the mathematicians and physicists have found the only efficient way of expressing "in the abstract" a thousand principles which furiously interest business men, military men and indeed Everyman when they take concrete form as statistics, patents, H-bombs and so on. Though "pure mathematics" is, admittedly, no more concerned with being useful than the "fine arts" are, it possesses a kind of trade-secret which the world finds more and more useful. The balancing-feats that go on around the = sign are ways of proving something absolutely, not just by default; and more and more different kinds of people now need *that way* of proving something. In short, the periodical and book printers are finding "formula work" turning up in more and more publications that are not strictly "scientific". Printers who even thirty years ago could reach some status as periodical houses with only the most elementary knowledge of mathematics are now no longer asking how much they might have to know about the subject before the next request-for-quotation comes in.

That once rhetorical question is now real enough to be re-phrased. What, they now ask, is the *least* that the modern printer is *expected* to know about "formula work" and the like? The University Presses are of course expected to know everything. They must not only understand mathematical setting; they have to understand *mathematicians* too, and be prepared to argue with them about the shape and significance of proposed new symbols for brand-new concepts. Certain other houses are expected by reputation to understand what even the most distinguished formula-jugglers are getting at, and the size of their mathematical equipment is one proof of their competence.

But that does not answer the question which the masters of ordinary middling-to-large printing offices are asking themselves today and discussing with their men. What is the new *minimum* of knowledge and competence required for mathematical setting? What, for instance, would H.M.S.O. expect in the way of "understanding" from the relatively large number of houses to which it entrusts work *involving some* formula work?

That would be the best test, because Her Majesty's Stationery Office must know for a fact, by bitter and repeated proof, the one thing that the O.U.P. and the C.U.P. *do not* know about Mathematical Composition. They do not know, they even politely refuse to believe, *what mistakes* are likely to be made when non-specialist printers tackle even fairly simple formula work. We may agree with Mr. Toad that

> "The learned men of Oxford
> Know all there is to be know'd . . ."

—except certain hard facts about loss of time and cost through misunderstandings which would simply never have arisen in the first place, in those higher reaches.

We hope that this RECORDER and its supplementary Mathematical Sorts List will be useful items for reference, above all in those offices where they will be needed less as manuals than as a reminder of what any printer is able to do profitably, with work involving occasional formulae, when he has 'Monotype' machines and some working knowledge of the Silent Language.

Author's Acknowledgements

I AM indebted to several persons who have helped me in preparing this paper although I am entirely responsible for the views expressed. The manuscript has been read by Mr. D. H. Sadler, Superintendent, H.M. Nautical Almanac Office, and by Dr. E. T. Goodwin, Superintendent, Mathematics Division, National Physical Laboratory, and by Mr. F. W. J. Olver of the same Department who has kindly read the proofs and has given me valuable advice. I am also indebted to Mr. P. R. Barrett who has read the proofs, and Sir Cyril Burt for his constructive observations. I have had many discussions with Mr. F. W. Latham of H.M. Stationery Office Press, Harrow, who has thus helped me to solve some of the obscure technical points.

At the present time no one would attempt to write a paper on the setting of mathematics without consulting *The Printing of Mathematics* (T. W. Chaundy, P. R. Barrett and Charles Batey, Oxford University Press, 1954). That book has done much to codify the style of mathematical setting and to expound the problems of mechanical composition. It gives admirable advice to authors and deals so well with the problem of punctuation that I have not considered it worth including this subject in the RECORDER.

In preparing the Mathematical Sorts List Mr. Brooke Crutchley has given me information which has helped me to identify some of the matching signs. It has not been possible to give names to all the 'Monotype' mathematical signs, but several people have helped me, including Dr. T. W. Chaundy, Mr. P. R. Barrett, and Dr. Mandl of the Atomic Energy Research Establishment, Harwell.

Finally, the whole project would have been immeasurably more difficult without the active co-operation of Mr. Philip Wright of Messrs. John Wright and Sons Ltd., who have printed this issue of the RECORDER and the Sorts List. The printer has managed to keep track of about 1,500 special sorts which have had to be associated with their correct matrix number and has at the same time maintained a high standard of composition. I am also personally grateful to Mr. Philip Wright for the technical advice and assistance he has given me.

A. P.

SETTING MATHEMATICS

A GUIDE TO PRINTERS INTERESTED IN THE ART

BY ARTHUR PHILLIPS

THE MATHEMATICAL MANUSCRIPT

The Author's Responsibility

SOME of the examples shown in this paper are not well set because the author has not chosen the most convenient notation for the printer. Co-operation between the author or editor and the printer is essential because the printer cannot be expected to have the mathematical knowledge that will permit him to amend a manuscript to a more convenient form without altering the mathematical sense.

No printer can be expected to set mathematics accurately from badly prepared copy, and an understanding of the printer's difficulties will aid authors in providing clear and satisfactory copy.

An author has the choice of either writing or typing formulae. Errors are easily made by the printer in interpreting an author's typescript because an ordinary typewriter has not sufficient signs or variability in spacing. Writing formulae in manuscript permits flexibility of expression, but the author should remember that the printer has no context, unless he is also a mathematician, to guide him in the identification of characters, and that errors in setting will be greater than in plain English unless the copy is carefully written. The capital letters *C*, *K*, *M*, *O*, *P*, *S*, *U*, *V*, *W*, *X*, *Y*, *Z* can be easily read as lower-case, and *O*, *o*, o; *e*, *l*, 1; *x*, ×, are easily confused unless care is taken in their size and formation. Typescript capitals and lower-case will not be confused, but a serious disadvantage of typing is that there is no difference between the size of a value and its exponent or suffix.

A typewriter which can be adjusted half a line at a time allows ordinary figures and letters to be typed in the correct place for superiors and inferiors, but not, of course, in the correct size. There is a dual-unit typewriter with one carriage and two keybanks; the additional keybank can be equipped with mathematical sorts which can be selected to suit the requirements of the user. It is usual to include selected Greek capitals, Greek lower-case, and superior figures on the auxiliary keybank together with brackets, sigma and two-line integral sign. This typewriter removes confusion arising from identifying Greek letters but leaves it to the printer's experience to interpret the size of characters which should be used. There is no doubt, however, that carefully written mathematical formulae will be acceptable to the printer.

Badly written manuscript Greek letters are easily confused with similar roman letters especially when written as suffixes, and there is a similarity between some Greek characters that can lead to errors. Unless it is absolutely unavoidable an author should not use the Greek nu (ν) and upsilon (υ) in the same manuscript. The Greek iota (ι) and omicron (o) being similar to italic are rarely used. Greek letters should be indicated, where necessary, by a red underline or by a pencil cross under the letter. The name of the Greek letter should be shown in the margin the first time it appears in the manuscript with an occasional repetition of the name if there are long intervals in its appearance. In a continuous series of equations containing many Greek sorts it will obviously be unnecessary to indicate every appearance of a Greek character. There is usually no need for these precautions when Greek characters are typed. The following sorts are those most easily confused:

$\alpha\, a \propto$; $\gamma\, y\, v$; $\delta\, \partial\, d$; $\varrho\, \epsilon\, e\, l\, I$; $\zeta\, \xi$; $\eta\, n\, y$; $\kappa\, k\, K$; $\nu\, v\, u\, \upsilon\, r\, \mu$; $\varpi\, \tilde{\omega}\, \tilde{w}\, \pi$; $\rho\, p\, P$; $\sigma\, o$; $\phi\, \psi$; $\chi \times x\, X$; $\omega\, w\, W$.

Notation

Signs, symbols, letters and figures are used conventionally by the mathematician as parts of speech; printers need to understand some of this mathematical grammar if they are to set mathematics correctly. Signs can be used as verbs to describe an operation; thus "from 5 take away 3" is written in mathematical shorthand as $5-3$. The plus and minus signs are also used as adjectives to describe the condition of a number as either $+5$ or -5. Printers who undertake mathematical setting to any extent will need a copy-preparer who must know the more important mathematical conventions.

The copy-preparer must be able to read the author's manuscript and know the correct size, position and spacing of all the various components of the mathematical expressions. The compositor, guided by the copy-preparer's marks, does not need such a detailed knowledge, but the greater his experience the less will be his errors in setting.

The advance of mathematics has been dependent upon the development of notation, for a simple and accepted notation is necessary to express and develop mathematical reasoning. Figures and letters are used with equal freedom by the mathematician. Letters are used to indicate an equality or other relation which is true for any value given to them. Letters are also used for unknown values and values between certain limits, and for special physical or mathematical constants. Letters or figures occurring in formulae must not be transposed because this may alter the meaning.

The most important fact for the compositor to remember is that figures and letters in mathematics have not only a horizontal position value but also a vertical value. For example, S^n and Sn are two very different quantities.

Authors should make every effort to use commonly accepted symbols and a notation which will assist the printer, and should follow the recommendations given in The Royal Society's publication *Symbols, signs and abbreviations recommended for British scientific publications.*† It may be necessary to use script characters for real and imaginary parts of complex numbers, but German and script characters should be avoided where possible. German books on vector analysis are often printed without Fraktur characters, and it is suggested that authors of mathematical books in English should be content with bold roman characters or bold sans serif for vector quantities. Authors should be particularly wary of using turned sorts for special symbols or values, for the compositor may easily turn them back after proofing, thinking they were inverted by mistake.

Dotted and barred characters are conventions, used, for example, for differentiation with time and for mean values. If many of these characters are required the printer should obtain special matrices and not try to make up barred sorts by using rules. An author may be willing to use primes instead of barred characters, but the requirements of the manuscript will have an important influence on the choice of type face (*see* p. 10).

Whilst it would be unreasonable to ask the author to forgo an accepted notation because it was difficult for the printer, it is in the author's interest that the work should be set at minimum cost; co-operation with the printer in the selection of the simplest notation will reduce the cost of setting.

Numbering formulae

For easy textual reference it is the usual practice in this country to number each formula by a number in parentheses full out to the right. For appearance's sake it is better not to use leaders between the formula and the number. Authors can help to reduce the cost of setting by giving a number to each formula whenever it is practicable thus to avoid the use of braces. Numbering should be done by a decimal notation as used in this paper, but where there is a closely related group of equations a brace may be used. The decimal system has the advantage that formulae can easily be added or deleted at galley-proof stage without extensive re-numbering.

Footnotes

The asterisk (*) is frequently used in mathematical notation; some authors may feel inclined to extend their notation to include the dagger (†) and double-dagger (‡). This desire should be resisted, for although "$\Sigma s^{\dagger}_{\lambda\tau} c^{\ddagger}_{\mu\tau} = \delta_{\lambda\mu}$ where $\dagger = E, D$ or P and $\ddagger = S$ or T" avoids an impossible number of exponents, it leaves no footnote references except § ‖ ¶. The normal practice is to leave the asterisk for the author to use in mathematical formulae if he so wishes and to start the footnote references with the dagger. American authors frequently use superior numbers in parentheses [(2)] for footnotes or references. These are satisfactory in the text but may be ambiguous if used after formulae. Under no circumstances should superior numbers be used without the parentheses, for this is certain to lead to their being confused with exponents.

† The Royal Society recommend roman for all "operators" and constants, but it is common practice to retain italic for the "d" in dy/dx.

THE PRINTER'S PROBLEM

Common Mistakes

There are several requisites for successful mathematical setting. Both the 'Monotype' operator and the compositor need an elementary knowledge of mathematical notation. The author's copy must be legible and without ambiguity. The printer must give some thought to the problem of mathematical setting before undertaking such work, for it is all too easy to underrate the extent of the planning and equipment required for mathematical composition. Until recently there has been very little information available on mechanical composition of mathematics and any printer who has rashly taken on this work has found it a costly experience. The following pages are an effort to provide some of the detailed "know-how" of mathematical setting, but before we embark on the detail it would be as well to look at the common errors of printers who have set mathematics without sufficient forethought.

Small alterations of position and size of figures and symbols can make nonsense of mathematical expressions and extensive corrections are often necessary because compositors lack knowledge of the correct notation and repeat the same errors. Here are the most common mistakes:

1. Inferior and superior sorts are set in the wrong size and type face and are positioned incorrectly.
2. Brackets are the wrong size and weight.
3. Fractions are not placed correctly in the formulae and are set in the wrong size.
4. Vertical spacing is badly arranged. This may be due to casting sorts on the wrong body, use of sorts of mixed alignment, or lack of knowledge of the correct alignment.
5. Horizontal spacing is incorrect, the compositor being unaware of rules of spacing that are peculiar to setting mathematics.
6. The correct signs and symbols are not recognized by the compositor.
7. Roman is used instead of italic and the compositor does not recognize the correct Greek letters.
8. The printer uses a mixture of special sorts in different sizes; any one size might be acceptable, but a mixture is intolerable. Also some printers not possessing the correct sorts endeavour to make them by mutilating other characters.

The compositor should not be satisfied with attaining only mathematical accuracy, but should strive for an elegant and easily read arrangement. Even when a good deal of time can be devoted to copy preparation it is not wise to give the compositor too many instructions which may confuse rather than assist. A simple code of copy-marking must be used to avoid mistakes.

Mathematical Equipment

Mathematics is usually set on a 'Monotype' keyboard, cast, and then made up correctly by hand. The amount of handwork can be reduced by putting as many signs, symbols and figures as possible into the 'Monotype' matrix case and by providing sorts cast on the correct body and set width for easy justification.

Many special sorts are required for setting mathematics; some additional 'Monotype' equipment is also necessary for the book printer who intends to take up mathematical printing. Before making any decision on equipment the printer should have a clear idea of the class of mathematics he intends to set and the type face and size he will use.

Mathematical setting can be broadly grouped into three classes: (*a*) the school text-book containing the elements of various branches of mathematics; (*b*) mathematical workings in general scientific periodicals and the more advanced text-book; (*c*) the advanced mathematical monograph and examples of higher mathematics in advanced scientific publications.

An 11-point Modern Series 7 matrix-case arrangement

Units	NI	NL	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
5	ι	j	i	‘	’	.	,	l	i	□	l	t	!	·	′	′	|	1
6	f	θ	1	2	3	4	5	-	j	□	b	i	0	[	]	I	(	2
6	q	v	1	2	3	4	g	/	f	:	½	j	0	;	{	}	)	3
7	x	k	a	α	π	μ	r	s	t	□	e	c	r	s	z	″	ω	4
8	ϵ	τ	;	p	I	z	c	e	g	b	o	q	v	n	z	s	∂	5
9	ζ	ξ	w	θ	λ	δ	ρ	α	β	I	√	κ	γ	η	F	P	J	6
9	¼	½	χ	−	±	+	g	o	a	□	a	d	h	x	*	–	L	7
9	¾	⅓	1	2	3	4	5	6	7	k	8	9	0	y	T	σ	C	8
10	?	ﬂ	ﬁ	y	v	q	b	h	n	p	ψ	ϕ	π	V	Q	G	A	9
10	m	ﬂ	ﬁ	⋮	x	u	p	k	d	n	u	S	m	R	O	E	B	10
11	ﬀ	ﬀ	J	Q	Z	Z	C	J	Y	X	U	N	μ	S	K	H	D	11
12	Γ	ω	ℐ	Σ	P	L	F	T	w	Δ	Ξ	∇	C	G	L	O	M	12
13	ϖ	V	Q	O	G	E	B	A	w	F	E	P	R	T	Φ	B	Λ	13
14	ℛ	Θ	ﬄ	R	U	Y	N	D	m	ﬃ	Ω	D	V	A	U	𝔄	w	14
15	→	Ψ	Π	ﬄ	X	M	K	H	m	ﬃ	H	N	K	∞	X	Y	□	15
18	⩽	⩾	∛	…	<	−	÷	W	×	~	=	±	M	W	+	—	□	16
	NI	NL	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	16th Row only
	H		HM							HN	HM					MN	HMN	

Type size. School text-books are frequently set in 9-point. This practice is not advocated; it is adopted by the publisher for economic reasons, but it would be of great assistance to both the student and the printer if a larger face were used. Mathematical appendices to general scientific works are frequently set in 9- or 10-point. For books on higher mathematics the choice rests between 11- and 12-point.

If formulae are set in 12-point, then literal inferiors and superiors can be set in 6-point. For higher mathematics 11-point offers most advantages; suffix and index literal values can be set in 5½-point, and if necessary 6-point sorts can be cast on a 5½-point body. The advantage of 11-point is in the ease of making up two-line fractions when the addition of a medium on 2-point rule makes the depth of the two lines up to 24 points.

Moulds. The use of 11-point makes it essential to have a 5½-point mould, and the possession of a 6½-point mould is a great advantage, for sorts and spaces can be cast for the case. The 6½-point material placed above and below the 11-point line makes up to 24-point, thus giving easy vertical justification with integrals. When 12-point is used for formulae a 26-point mould will provide spacing material for two-line fractions and will enable braces, parentheses and signs to be cast on a 26-point body for ease in make-up. A 7-point mould will serve the same purpose in the make-up of integral limits for 12-point formulae as does the 6½-point mould in making up 11-point formulae.

A short-rule mould is essential to provide 2-point rule for two-line fraction bars. Although rules can be cut from lengths and then the ends squared in the mitring machine, this is a laborious and inaccurate process. With the short lead and rule mould, rules can be cast to unit lengths of the normal set of the text type to a maximum length of 36 points.

Letter-spacing and unit-adding attachments. It is frequently necessary to use a 2- or 3-unit letter-space, either by the unit-adding or the letter-spacing attachment. This problem is discussed under letter-spacing (p. 13).

The mathematical matrix case

Many printers are using the 16×17 matrix case for mathematical setting. This provides 272 matrices or 266 characters after allowing blanks for spaces. If the 16×17 matrix case is used with a 13-row keybank some adjustment must be made to the key arrangement in order to accommodate 266 characters. The usual method is to make use of the justification keys for characters. The characters which are placed on the top row of justification keys must occupy succeeding *numbered* rows in the matrix case. One layout uses fourteen characters of the "O" row from top to bottom in place of the top justification keys, which in this mathematical layout become character keys. When justification is required the 0·0005 punch or the 0·0075 punch is operated by a separate key; it must be depressed simultaneously with the correct character key, which on the normal keybank is a justification key.

The keybar frame can be made up to give any desired character in place of the lower justification keys. Additional keys can also be obtained by making the space key overhang the keybank frame.

The standard 14-row keybank provides a maximum of 266 characters, and to save changing keybars it is sometimes useful to have a few spare positions.

Additional keys can be made available by placing the superior and inferior figures in the corresponding rows to the normal figures but in the 6-unit position. In the illustration shown (Messrs. John Wright & Sons Ltd.) to obtain superior or inferior figures the operator has to depress the opening or closing parenthesis key simultaneously with the normal figure key.

Because mathematics requires many additional sorts it is often difficult to use even a six-alphabet arrangement. The matrix-case arrangement illustrated provides five alphabets, viz. roman and italic upper and lower case and small capitals. It is an advantage if small capitals can be omitted from the matrix case without loss to the typographical appearance of the work and so leave more room for special sorts. The aim of the keyboard operator should be to provide as many sorts as possible to save hand-setting, and to achieve this end it is necessary to make frequent changes in the matrix-case arrangement.

Precision Make-up of Mathematics

For ease in setting mathematics the compositor must be provided with legible copy and adequate material, and he must possess some knowledge of the subject and interest in his work. But, though all these things are necessary, one of the main factors in ease and speed of setting is precision make-up of mathematical expressions. This requires careful planning and the co-operation of keyboard operator, caster attendant and hand compositor.

It has already been mentioned that a $5\frac{1}{2}$-point mould is necessary to provide superiors for 11-point body type, and that a $6\frac{1}{2}$-point mould is required for spaces and characters that appear with the 24-point integral sign. The use of 11-point type permits the easy make-up of two-line fractions. All these adjustments are in the body depth, but for real precision it is necessary to control the unit width of all material that will be made up to more than one line in depth. One method is to restrict the unit width of 6-point and $5\frac{1}{2}$-point sorts which will be used as superiors or inferiors. This reduces the variety of spaces required to justify them in depth and also makes it easy for the operator to tap sorts or positive spaces of equivalent value in the text type. These will be changed by the compositor for the correct superiors or inferiors and the correct space to give justification in both depth and width.

Messrs. John Wright & Sons Ltd., who have set this RECORDER, have extended this system of equivalent sets in the following manner. They cast all sorts intended for use in mathematical setting, regardless of point size, on bodies having set widths of whole numbers of units of $10\frac{1}{2}$ set (the set of 11-point Modern Series 7, in which the bulk of their mathematical composition is done). Justification is greatly simplified, not only because the keyboard operator can tap a substitute character of precisely correct set, but even more because the compositor can so readily and so accurately justify superiors with inferiors immediately below them. This operation has been still further simplified and systematized in the following way. The mathematical sorts cases are plain open cases which have been filled with small metal boxes in 10 different colours. Santype coloured plastic boxes, which were not available when John Wright & Sons started their system, might now be advantageously employed. Each of the 10 colours represents a figure from 1 to 10, and the sorts are placed in boxes of which the colour represents at a glance the unit width of the characters it contains. It will, therefore, readily be seen how much the compositor's work is simplified by being able to justify with certainty and not by experiment.

Accurate control of casting is also necessary, with micrometer checks on set width of sorts and meticulous care to ensure correct alignments. This care must be exercised both in casting sorts for case and in casting composition. Sorts may be changed by hand for equivalent unit values at any time, and it is therefore most important that the caster attendant should not alter the micrometer wedge to suit the type measure but that all adjustments should be made to give accurate set.

Choice of Type Face

Mathematical signs, and superior and inferior letters and figures, have been mainly designed to work with Modern Series 7. By present standards this type is not considered to be among the very best designed book faces. If, however, more aesthetically attractive types are used for mathematics some difficulties are likely to occur, the extent of the problem depending upon the requirements of the manuscript. When a wide range of barred and dotted characters is required it is advisable to use Modern Series 7 or Old Style Series 2, for these type faces provide a large assortment of special characters. Another advantage of Modern Series 7 is that few italic characters are kerned and there is therefore less necessity to letter-space ascending and descending sorts that are followed or preceded by superiors, inferiors or parentheses (cf. letter-spacing, p. 13).

This does not mean that Modern Series 7 or Old Style Series 2 are the only faces that can be used for advanced mathematics; Greek characters, superiors and inferiors, signs and symbols work very well with Imprint, which at present only lacks some characters with mathematical accents. The following type faces are those most frequently used for mathematical books and periodicals:

Modern Series 1	Imprint Series 101
Old Style Series 2	Baskerville Series 169
Modern Series 7	Times New Roman Series 327

In 1950 The Royal Society Consultative Committee for Co-operation with Printing Organizations recommended that the choice of type faces for mathematical works should be restricted to Modern Series 7, Imprint Series 101 or Times New Roman Series 327.

Authors, typographers or publishers should not expect a printer to set mathematics in *any* type face. The printer's mathematical equipment is usually related to one or two faces with a preference for either 11- or 12-point; his advice on the choice of type ought therefore to be accepted.

All literal values in mathematics, whether in the body size, superiors or inferiors, should be set in italic; this is a conventional method of presentation and helps the reader to recognize textual references. Roman is frequently used for operators and constants.

Greek type for mathematics

There are five factors which influence the choice of Greek type to work with roman characters: they are x-height, weight and stress, alignment and set. It is important that the x-height and weight of the Greek face should be comparable to that of the roman. The x-height of Greek is less than that of most roman faces. This means that the non-ascending characters, $\alpha\gamma\epsilon\eta\iota\kappa\mu\nu o\pi\rho\sigma\tau\upsilon\chi\omega$, are smaller than the corresponding roman characters and that the ascending lower-case, $\beta\delta\zeta\theta\lambda\xi\phi\psi$, and all the capitals will be of similar size to the roman. There must therefore be a compromise if Greek and roman founts are to be used together.†

Fortunately the x-height of the Greek face is not so important as at first sight it might seem. Providing the alignment of the two faces is good, the smaller x-height of the Greek is acceptable because of the different formation of the character compared with the familiar

† This does not apply to Times Greek Series 565, 566, 567 which have been specially cut to match Times New Roman or to Series 473 which has been specially cut to work with Baskerville Series 169.

roman alphabet. We would not accept the juxtaposition of roman and italic characters with a similar variation.

A table on page 32 shows the alignment and set of Greek and Fraktur type and the faces normally used for mathematical setting. In most cases a Greek face the same size or one point larger than the italic face is suitable.†

Variations in alignment of more than 0·002 inch will be noticeable if the two types are used in the same matrix case; if the difference in alignment is too great the Greek can be cast separately to align with the roman and inserted by hand, but this will increase setting costs. When it is necessary to set together types which align within the 0·002-inch tolerance but differ in set, then the sorts required can usually be placed in an equivalent unit row in the text matrix case; for example, 8 units of $8\frac{1}{2}$ set are approximately equal to 7 units of $9\frac{1}{2}$ set.

The choice of a text type must be influenced by the availability of a suitable Greek with a similar alignment if setting is to be done at minimum cost.

Inclined Greek is normally used for mathematical values which are set in lower-case, for the slope is similar to the italic used for literal values. Upright Greek is used for Greek capitals. The only Greek capitals which differ from the roman are ΓΔΘΛΞΠΣ ΥΦΨΩ. The remaining thirteen Greek capitals are rarely used in formulae, being indistinguishable from similar roman capitals.

The Greek founts normally used in mathematical work are Series 91 (inclined), Series 90 (upright), Series 106 (Porson), and Series 472 (specially cut to work with Modern Series 7). No author or publisher can expect a printer to be equipped with special barred and dotted characters in all these faces, nor to have Greek sorts cast to special alignment for individual jobs. Series 90 and 91 offer some advantages, but a greater number of mathematically accented characters have been cut in Series 106 than in any other Greek face.

New Hellenic Greek Series 192 has no stress and, whilst excellent for Greek text, is unsuited for mixing with book faces. The x-height of Porson is slightly less than that of Series 90 and 91 and the alternative alpha must be used (matrix 484), for the normal alpha is too much like an italic lower-case "*a*". The alpha of Series 91 is similar to Porson and either Series 90 alpha or matrix 483 must be used instead. Series 92 is used for bold Greek characters.

Greek superiors (L29) and inferiors (L30) are available in 11-point and the lower-case in 12-point.‡ Series 91 is available in 6-point for 12-point inferiors and superiors. It may be necessary to adjust the alignment of the 6-point to range with other superior characters. The set widths can be restricted and an equivalent character keyed to be exchanged by hand (cf. precision make-up). Porson $5\frac{1}{2}$-point can be specially cast for use as superiors to L29, thus providing superiors to superiors.

Mathematical signs, figures and peculiars

Signs. These are cut in various weights and face sizes and with different alignments. A printer who wishes to purchase matrices of mathematical signs should choose the sizes and weights that will work well together. The selection shown in the separate Sorts List has been chosen with this object. Signs should also be of a similar weight to the text type which the printer intends to use for mathematical setting.

Figures. Lining figures (e.g. F.340 Imprint) are used for formulae in most mathematical text-books. Old Style non-ranging figures are often employed for monographs on advanced mathematics. Non-ranging figures have the same form of lining as ordinary lower-case; superiors and inferiors work equally well with them as with lower-case (e.g. they have been used here for all correctly set examples); ranging figures (F.60, F.61 or L87, L88) are used for superiors and inferiors.

(F.340) 1234567890 (Imprint, Series 101) 1234567890

Script characters. It is necessary to use rectangular body script in order to bring suffixes and adjacent characters close up. Rectangular body script capitals are available to work with 8-, 9-, 10- and 11-point Modern Series 7, and these can also be used with Imprint, although the 8-point must then be cast to a different alignment.

Peculiars. There is a common trend in all branches of mathematics to extend the notation to include all kinds of special characters. This extension takes two forms: the adoption of characters for permanent physical or mathematical concepts or values, and the ephemeral use of letters in an individual paper. Thus $\pi\ \Pi\ \Sigma\ \epsilon\ \partial\ \hbar$ all have a commonly accepted use, but authors may ask the printer to provide a variety of characters that are their own invention. The Sorts List shows a wide range of accented characters. It is hoped that these will be sufficient to satisfy most authors and that it will be an exceptional need that requires the cutting of a new punch.

† Porson Greek Series 106, 11-point, is used in the examples in this issue of the RECORDER.
‡ For further information on superiors and inferiors see Mathematical Sorts List.

GUIDE TO MATHEMATICAL COMPOSITION

Spacing

THE compositor's first reaction to the word "spacing" is to think of word spacing, then line spacing, and perhaps letter-spacing as an afterthought. In mathematical setting letter-spacing is most important and it has therefore received special attention to both the means and application. Alignment, which might also be considered under spacing, has been dealt with either under the appropriate mathematical notation or where technical difficulties arise.

Spacing between lines

There must be sufficient space between formulae to separate the equations, and the eye must be able to follow each line clearly and without interference from adjacent lines. A half-white between lines of displayed formulae is often sufficient, but there are many occasions when complicated expressions require a full white to give adequate separation between lines. There should be at least a half-white above and below single-line displayed formulae; groups of formulae may require more space. Sometimes it is difficult for the compositor to know when the author intends formulae to be displayed; formulae may occur every two or three lines and in these cases readability must have prime consideration. As a general rule, all except the relatively unimportant formulae should be displayed. If these can be made sufficiently compact by using the solidus, then text and formulae should run on, providing the formulae are not broken by turning over. If fractions in the formulae make two lines in depth, then rather than make the text spacing uneven, it is better to display the formulae. Displayed formulae will usually be referred to in the text and frequently these mathematical references are easier to read if the text is leaded, but the appearance of leading should not be obtained by increasing the body depth. If the text type is cast on a body larger than normal then either the formulae must be cast separately, or all special sorts cast on the larger body. There would also be difficulty in making up two-line fractions; it is therefore advisable to use leads if the inter-linear space must be increased.

Horizontal spacing

The spacing between signs, letters, symbols and figures is partly dependent upon the Style of the House. It is not sufficient for spacing to be such that formulae are mathematically correct; the spacing must be arranged so that the various components of the formulae are easily readable and are correctly associated with each other.

The keyboard operator must on no account use the variable space in setting formulae except as a means of centring a line. Positive spaces should be used in a uniform manner dependent upon the set width of the various signs.

Formulae longer than one line

It is necessary to break formulae that are too long for one line. The length of formulae should have an influence upon the choice of format, it being wise to increase the page width if many of the formulae would otherwise require two lines. When a break is necessary a suitable place must be chosen. It is always safe to break at $= > \simeq$ signs or between two major brackets. The turn-over line should be indented a sufficient amount to bring it beyond the equal sign in an equation, although in a narrow-column periodical this may be impossible. If the break comes at the $= > \simeq$ sign, then the sign should appear only at the beginning of the turn-over line. If the break occurs at $+ - \times \div$, the sign should be at the end of the first line and repeated at the beginning of the turn-over line. If the break is between brackets, then the $\times$ sign should be added at the end of the first line and repeated in the turn-over line unless its omission has already been agreed upon with the author.

Some mathematical printers are satisfied with the sign at the commencement of the turn-over line without the repeat at the end of the first line; printers should decide their own House Style in this matter (1.1).

$$f(x) = s\left\{1 + \frac{1}{PD}[E_1(l+x_2) - E_2(l+x_3)] \times \right.$$
$$\left. \times [4E_2(1+x_3) + 4E_3(1+x_4)]\right\} \qquad (1.1)$$

Sequences of equations

Equations should normally be centred on the measure, but connected sequences of equations should have the equal signs ranging vertically unless there are special circumstances (1.2–1.5).

$$C_p = \tfrac{1}{2}C_c \qquad (1.2)$$

$$P_t = -\frac{C_c}{4} + \sqrt{\left(\frac{C_c^2}{16} + \frac{C_c^2}{400}\right)} \qquad (1.3)$$

$$= \frac{C_c}{4}\left\{\left(1 + \frac{1}{25}\right)^{\frac{1}{2}} - 1\right\} \qquad (1.4)$$

$$\simeq \frac{C_c}{4} \times \frac{1}{50} = \frac{C_c}{200} \qquad (1.5)$$

In the last equation the $\simeq$ is ranged with the equal signs; this is done to keep the right-hand terms which have been approximated all to the right. Short equations may be doubled up on the measure when space and mathematical sense permit (1.5, 17.1, 17.2), but when formulae on the same line are not related they should be separated by a 3-em space.

Letter-spacing

It is not possible to give invariable rules for unit-spacing. It is comparatively easy to see when formulae are incorrectly spaced, but difficult to provide rules which can be followed in every circumstance. The legibility and general appearance of displayed formulae can be improved by careful letter-spacing.

The unit-spacing between figures, signs, and numerical or literal values is obtained by using either unit-adding and positive spaces or by letter-spacing and positive spaces, or by the use of all positive spaces when there is a 3-unit row in the matrix case. The unit-adding attachment enables the addition of either 1, 2, or 3 units to signs or other characters. The amount of space that could be added depended at one time upon the size of the character; the combined-spacing mould now removes any limitation previously imposed by matrix size. If the unit-adding attachment is used a caster wedge is required for every change in the number of units added, and the amount to be added must therefore be decided before setting is commenced.

The formulae in this issue of the RECORDER have been set with the addition of a uniform 3 units where letter-spacing has been necessary. If the letter-spacing attachment is used instead of the unit-adding attachment, the additional space is obtained by the justification wedges. The letter-spacing attachment requires less equipment but is somewhat slower to operate. Owing to difficulties in justification it is not feasible to add a varying number of units to different characters in the same line. It is best to limit the added space to either 2 or 3 units, which, with positive spaces, will be used consistently throughout the book.

The space added will always appear on the left of the character and the space is cast on the same shank as the character. This limitation is very important to the mathematical printer, for in machine composition the only way space can be added to or taken from the right-hand side of a character is by casting from a matrix that has been specially struck in the correct position. Kerned sorts can be cast for case with the kerns on the right by reducing the set width and adjusting the set-wise position of the matrix.

In (2.1, 2.2) the same formula has been set in Imprint and Modern Series 7. All seven italic characters are kerned in Imprint and ten letter-spaces are necessary. With Modern Series 7 italic only two characters are sufficiently kerned to require letter-spacing and six letter-spaces have been inserted; the 3-unit space after p_2 in (2.2) is not required because of a kern on y but to associate the suffix with p. As an alternative to letter-spacing, the matrix case could be made up with two matrices of the offending character: one in the normal position and the other in a row of higher unit value. The additional units will appear on the left-hand side of the character. It will be seen from this example that Modern Series 7 italic has advantages over the more attractively designed Imprint italic when used for mathematical setting.

$$\overset{3}{(}\overset{3}{f}{}^2+p)\overset{3}{(}p_2\overset{3}{y}+\overset{3}{f})\overset{3}{(}g+5g)+\tfrac{1}{2}\overset{3}{y}+4\overset{3}{f}+\overset{3}{i}(\overset{3}{l}) \qquad (2.1)$$

$$\underset{3}{(}\underset{3}{f}{}^2+p)\underset{3}{(}p_2\underset{3}{y}+\underset{3}{f})(g+5g)+\tfrac{1}{2}y+4\underset{3}{f}+i(l) \qquad (2.2)$$

Spacing the = sign. Eight units should *appear* each side of the equal sign; this allows for a slight break between the two halves of the equation. The full-face 18-unit sign is not recommended. The sign which is approximately 12-unit face centred on 18-unit shank (S3465) is more elegant and a 5-unit positive space

$$-\bar{V}(a,z) + iV(a,z) = \frac{\Gamma(\tfrac{3}{4}+\tfrac{1}{2}a)\,e^{-i\pi(\frac{1}{2}a-\frac{3}{4})}}{2^{-\frac{1}{2}a+\frac{1}{4}}\,\pi^{\frac{3}{2}}} \int_{(\xi_3)} e^{\frac{1}{4}z^2 t}\,(-1-t)^{\frac{1}{2}a-\frac{3}{4}}\,(t-1)^{-\frac{1}{2}a+\frac{3}{4}}\,dt \tag{3.1}$$

3 3 3 3 3 3 5 5 5 3 3 3 3 5 3 3 3 5 3 3 5

$$\psi \sim (\tfrac{1}{2}a+\tfrac{3}{4})\pi + px\left\{1 - \frac{\frac{2}{3}x^2}{(4p)^2} - \frac{\frac{2}{5}x^4+16}{(4p)^4} - \frac{\frac{4}{7}x^6+\frac{320}{3}x^2}{(4p)^6} - \frac{\frac{10}{9}x^8+544x^4+2944}{(4p)^8} - \dots\right\} \tag{3.2}$$

5 5 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3

$$E(a,x) \equiv k^{-\frac{1}{2}}\,W(a,x) + ik^{\frac{1}{2}}\,W(a,-x) = 2^{\frac{1}{4}}\,e^{\frac{1}{4}\pi a}\,e^{i(\frac{1}{2}\phi_2+\frac{1}{8}\pi)}\,D_{-ia-\frac{1}{2}}(x\,e^{-\frac{1}{4}i\pi}) \tag{4.1}$$

3 3 5 5 3 3 3 3 3 3 3 3 3 5 5 3 3 5 3 5 3 3

can be added each side to give the correct appearance of an 8-unit space.

Spacing the ×, +, −, *and* ± *signs.* The rules for spacing these signs must to some extent depend upon the care the copy-preparer and compositor can afford to give to the work.

The amount of space to be added to a sign depends upon the actual width of the sign and the set of the cast sort. Plus and minus signs which are 18 units full-face are frequently used; these are too large and often too light. In every case signs with a 12-unit face on an 18-unit shank are preferable and have been used for all correct examples shown in this paper. The matrices for these signs (+, S3460; −, S3461; ±, S3462) are struck so that the sign is cast centrally on an 18-unit shank and the sign will show a 3-unit space on each side even when the shank is close up to a character. This is an advantage, for there are no occasions when the sign need be any closer to the value and additional letter-spacing can frequently be avoided. Signs and characters can also be cast centrally on a 24-point body and inserted by hand to save make-up time in setting two-line fractions in 11-point (3.2, 4.2).

The small figures under the formulae show unit values of the letter-spacing. In the numerators of the main fractions of (3.1, 3.2) the signs are close up (i.e. 3-unit appearing space); a 3-unit space has been added where indicated. It is not suggested that formulae will be unreadable if all ×, +, −, ± signs are set close up, providing the small-faced signs are used, but the proposed spacing rules will assist in reading the more complex formulae.

The following rules have been adopted in the examples (3.1, 3.2). When the plus and minus signs are used to describe the condition or direction of a value there is no additional space between sign and value. No extra space is added to the signs used in suffixes or exponents. Some formulae require a wider range of spacing to obtain a readable grouping; additional space is placed each side of the sign connecting major values, the signs connecting minor values being close up. (In two-line fractions 2-point leads are used which are slightly more than 3 units.)

Spacing between values. Normally when values are multiplied together they are set close together, but where there are long exponents to several values multiplied together, space must be inserted. Example (4.1) shows the use of a 3-unit space between the arguments a, x, and 5 units after the exponents $\frac{1}{4}\pi a$ and $i(\frac{1}{2}\phi_2+\frac{1}{8}\pi)$; also 3 units after the suffix $-ia-\frac{1}{2}$. A kerned D cannot be used because the counter on the minus sign prevents it going under the kern. The use of the full point to indicate multiplication should be avoided where possible except, for example (4.3), or for its customary use as in the series (4.2). It should then be a 9-unit character.

$$y = C_0\left(x + \frac{x^4}{3\cdot 4} + \frac{x^7}{3\cdot 6\cdot 4\cdot 7} + \frac{x^{10}}{3\cdot 6\cdot 9\cdot 4\cdot 7\cdot 10}\right) \tag{4.2}$$

5 5 3 3 3 3 3 3 3

$$U(a,x) = \tan\pi a\,.\,U(a,x) + \sec\pi a\,.\,U(a,-x) \tag{4.3}$$

3 3 5 5 3 3 3 3 3 3 3 3

$$f(x) = e^{\frac{1}{4}(\phi r-1)}\,\sin\pi(\tfrac{1}{4}a+\tfrac{1}{2}) \tag{4.4}$$

3 5 5 3 5 3 3

Trigonometric functions and similar abbreviations will normally take a 3-unit space before them when multiplied by literal or numerical values and will be followed by a 3-unit space (4.3, 7.2). When the exponent preceding the trigonometric abbreviation is large (4.4) it is necessary to increase the space to 5 units. It is sometimes necessary to insert a space before an exponent, particularly with a fractional exponent, and before or after parentheses (3.1, 4.3). Additional space is not necessary before a negative exponent.

In the integral calculus the differential "*dx*" and the partial differential "∂x" are used, and with similar values appear as one group of characters. The letter "*d*" is not a value by itself and is inseparable from its associated character; therefore when "*dx*", "*dy*" or similar pairs of letters prefixed by "*d*" or "∂" appear in the same group of characters they must be separated by a 3-unit space between each pair of letters (5.1, 21.1, 21.2).

In order to compensate for the space shown in the counter of parentheses and brackets of the body size it is necessary to add a 3-unit space (3.1) between the bracket and a term multiplying the values in the bracket. When large brackets are used a corresponding larger space must be allowed (5.3, 17.3, 18.4). A 3-unit space is also necessary before a colon, semicolon, or exclamation mark used as the factorial sign (17.3).

Fractions two lines deep in the body size, multiplied by single-line values, should be separated from that value by the equivalent of 3 units.

It is not always possible to show that values are multiplied together merely by setting them close together. Examples (4.3) and (**5.2**) are similar instances: in (**5.2**) $\sin\frac{q\pi}{2}$ is multiplied by A_{1q} and $\frac{x}{l}$; if the point were omitted it might appear that $\sin\frac{q\pi x}{2l}$ was the value. The full point must range with and be centrally between the two fraction bars (28.1) and not as (**13.1**)*. In (**5.2**) the full point need not be used, as the alternative (5.3) indicates. It would be much better, however, if the author rearranged the expression as (5.4).

$$V_z = \int_0^{2\pi}\int_0^{\beta}\int_0^{\alpha} zr^2 \sin\theta \, dr \, d\theta \, d\phi \qquad (5.1)$$

5 5 3 3 3 3 3

$$a = \sum_{q=1}^{2n-1} A_{1q} \sin\frac{q\pi}{2}\cdot\frac{x}{l}\cosh\frac{q\pi}{2k_1}\cdot\frac{s_1}{l} \qquad (\mathbf{5.2})$$

5 5 5 3 3 5 3

$$a = \sum_{q=1}^{2n-1} \left(A_{1q}\sin\frac{q\pi}{2}\right)\left(\frac{x}{l}\cosh\frac{q\pi}{2k_1}\right)\frac{s_1}{l} \qquad (5.3)$$

5 5 5 3 3 3 5 3 3

$$a = \sum_{q=1}^{2n-1} A_{1q}\frac{xs_1}{l^2}\sin\frac{q\pi}{2}\cosh\frac{q\pi}{2k_1} \qquad (5.4)$$

5 5 5 3 5 3 5 3

Suffixes and Indices

Suffix letters are used by the mathematician to identify different values represented by the same letter. Some authors are inclined to use too many suffixes, for it is easy to acquire this habit in writing. Manuscript containing many suffixes should be edited and the values replaced by additional letters. Authors should be particularly careful to avoid writing suffixes to suffixes. Example (**6.1**)* shows suffixes carried to absurdity; the printer would have been justified in returning this copy to the author to be rewritten with a different notation. Also the multiplicity of suffixes has led the printer to make errors of alignment that might otherwise have been avoided; the $r\theta_1 \log_e$ should align with p_2 and γ_w and the minus signs should be central on the depth of "p".

If several suffixes *must* appear to the same value they can be separated by commas. Example (**6.2**) shows (**6.1**)* reset with the same suffix letters and figures but with the suffixes separated by commas. The Greek gamma with suffix "*w*" is used to represent water at various temperatures, and the "*w*" does not seem necessary. It is appreciated that authors prefer to use letters which have by practice become associated with certain

$$\frac{\mathrm{T}_{\theta_1}}{\mathrm{T}_{\theta_2}} = \frac{\mathrm{p_2}_{\theta_1} - \mathrm{p_1}_{\theta_1} - \gamma_{\mathrm{w}_{\theta_1}}\, \mathrm{r}\theta_1 \,\log_e \dfrac{\mathrm{p_2}_{\theta_1}}{\mathrm{p_1}_{\theta_1}}}{\mathrm{p_2}_{\theta_2} - \mathrm{p_1}_{\theta_2} - \gamma_{\mathrm{w}_{\theta_2}}\, \mathrm{r}\theta_2 \,\log_e \dfrac{\mathrm{p_2}_{\theta_2}}{\mathrm{p_1}_{\theta_2}}} \qquad (6.1)^*$$

$$\frac{T_{\theta_1}}{T_{\theta_2}} = \frac{p_{2,\theta_1} - p_{1,\theta_1} - \gamma_{w,\theta_1} r\theta_1 \log_e(p_{2,\theta_1}/p_{1,\theta_1})}{p_{2,\theta_2} - p_{1,\theta_2} - \gamma_{w,\theta_2} r\theta_2 \log_e(p_{2,\theta_2}/p_{1,\theta_2})} \qquad (\mathbf{6.2})$$

physical qualities, but even so it is quite permissible to represent temperature by "$\tau, \theta, t, T, \Theta$". At first sight there appears to be some excuse for "p_1, p_2" which represent two values of pressure. However, the context reveals that the pressure is derived from two different sources and there would be less confusion in the mind of the reader if "q" were substituted for "p_2". Equation (6.3) is (**6.1**)* with the following substitution:

$$\theta_1 = \theta, \quad \theta_2 = \tau, \quad p_1 = p, \quad p_2 = q, \quad \gamma_w = \gamma$$

The result is now so easy to read that it is obviously tautological.

$$\frac{T_\theta}{T_\tau} = \frac{q_\theta - p_\theta - \gamma_\theta r\theta \log_e (q_\theta/p_\theta)}{q_\tau - p_\tau - \gamma_\tau r\tau \log_e (q_\tau/p_\tau)} \qquad (6.3)$$

Dotted Greek letters should not be used for suffixes if they can be avoided. The $\dot{\theta}$ suffix in (**6.4**) is not a readable notation and the author should write the expression in some other form replacing $\dot{\theta}$ and $\ddot{\theta}$ by other more readable symbols.

$$M = M_\theta \theta + M_{\dot{\theta}} \dot{\theta} + M_{\ddot{\theta}} \ddot{\theta} \qquad \textbf{(6.4)}$$

Setting suffixes

As with superiors and indices, suffix letters can be cast separately on a few selected unit widths for hand setting. By restricting the characters to a few different unit widths a considerable amount of hand-compositor's time can be saved (cf. precision make-up).

When several values are multiplied together and contain suffixes they should not be set close up, but a 3-unit letter-space should appear after the suffix to prevent it being associated with the following value. In (**7.1**)* the suffixes are incorrectly spaced and in (7.2) correctly spaced. Italic capitals in (7.2) are kerned to bring the suffix close to its associated value.

$$M = k_1 T_c \frac{d}{2} \sin\theta + C_p b_y d_y + Y_1 k_2 k_4{}^2 \qquad \textbf{(7.1)*}$$

$$M = k_1\, T_c \tfrac{1}{2} d \sin\theta + C_p\, b_y\, d_y + Y_1 k_2 k_4^2 \qquad (7.2)$$

5 5 3 3 3 3 3 3 3 3 3 3 3 3

Although it is always possible to cast italic capital letters specially kerned for insertion into expressions by hand, this may well entail a considerable amount of extra hand work in manuscripts where italic capital letters with suffixes occur frequently. In 11-point Modern Series 7 and 12-point Baskerville Series 169 full capital alphabets of italic matrices are available which have been so struck as to give kerns on the right-hand side of all the characters. The insertion in the matrix case of several of those most frequently occurring will, in setting some manuscripts, save the compositor a tremendous amount of handling.

The author should clearly indicate where suffixes both precede and follow a value; letter-spacing must be used to associate them with the correct value (17.4).

Ranging suffixes. There is sufficient beard on the main values for a suffix to appear in the correct vertical position if the shanks of the two letters range at the foot. Any effort to make the suffixes lower (**6.1**)* will be unsightly and cause the compositor a lot of unnecessary trouble. Inferior figures should be of similar weight and should range with inferior letters. Example (**8.1**)* shows a mistake made by a compositor when setting from typewritten copy where suffixes and values are the same size; (8.2) is the correct setting.

$$3({}^e l + \alpha^e t) + [{}^e\theta + \alpha^e(\theta + 90)] \qquad \textbf{(8.1)*}$$

$$3\,(e_l + \alpha e_l) + (e_\theta + \alpha e_{(\theta+90)}) \qquad (8.2)$$

Setting indices

The compositor must be able to recognize which values in the manuscript are indices (exponents) and must therefore be set as superiors. The exponential function "e" is usually followed by index values; either the roman (e), italic (*e*), or the Greek epsilon (ε) may be used. The abbreviation "exp" can be used instead, but this is never followed solely by superiors. If the form "exp" can be used, it is more readable (9.1), but mathematicians do not always find that this form of expression is convenient.

$$f(x) = \exp(-\kappa t \alpha_5^2/a^2) \quad \text{not} \quad f(x) = \mathrm{e}^{-\kappa t \alpha_5^2/a^2} \qquad (9.1)$$

Errors are often made in setting fractional indices which are mistaken for ordinary fractions. Either $5\frac{1}{2}$- or 6-point ordinary fractions or split fractions must be used for fractional indices. If the index value has a literal denominator a solidus should be used. Split fractions give a very compact expression, but some

$$\frac{r}{a} \leqslant e^{\frac{\pi}{2}} \qquad \textbf{(10.1)}$$

$$\frac{r}{a} \leqslant \mathrm{e}^{\frac{1}{2}\pi} \qquad (10.2)$$

$$\frac{r}{a} \leqslant \mathrm{e}^{\pi/2} \qquad (10.3)$$

$$df = v\,\mathrm{e}^{-v^2/(mvx)^2}\, dv \qquad (10.4)$$

$$df = v \exp[-v^2/(mvx)^2]\, dv \qquad (10.5)$$

$\frac{x-y}{a-b}$ then brackets must be placed round the numerator and denominator thus: $(x-y)/(a-b)$.

The solidus should not be used unnecessarily in displayed formulae, but there are occasions when it will permit a more elegant setting. It is useful for a single-value fraction in either a numerator or denominator of a main fraction. Example (**14.1**) shows how the solidus has avoided a fraction four lines deep, but (14.2) is a better form of (**14.1**); it avoids the ugly $\frac{1}{c}$, and $\sinh \frac{1}{2}c(1-x/l)$ is better than $\sinh c(1-x/l)/2$. The author should consider how his expressions will look when set and put them in the most readable form.

$$\sigma_2 = \frac{Pl}{10Ad} + \frac{1}{c}\frac{\sinh c(1-x/l)/2}{\cosh c/2} \qquad (\mathbf{14.1})$$

$$\sigma_2 = \frac{Pl}{10Ad} + \frac{\sinh \frac{1}{2}c(1-x/l)}{c \cosh \frac{1}{2}c} \qquad (14.2)$$

If a horizontal fraction bar is used in a fraction that makes more than two lines deep, then the main fraction can be separated by a slightly heavier fraction bar to assist the reader to group the values (15.1). Sometimes the three-line fraction can be avoided by using the solidus.

$$\frac{c}{y_2} = \frac{m(x_1+y_1)(x_2+y_2)}{\frac{3}{n_1}+\frac{5}{n_2}+\frac{7}{n_3}\left(1+\frac{y_1}{x_1}\right)} \qquad (15.1)$$

The following expressions (**16.1**–16.12) are shown here to give a clear impression of the correct and incorrect use of the solidus. In most instances it is the author's thoughtless use of the solidus which has led the printer to set an inelegant expression.

Example (**16.1**) would be correct if used in run-on text, but (16.2) is better for displayed formulae. There are a number of errors in (**16.3**)*; the second and fourth $\pi/2$ are indices, the first and third are not. A compositor who is used to mathematical setting would recognize that the $\pi/2$ which follows the exponential function (e) is an index value (exponent). In every case where the numerical value is in the denominator the fraction should be set as $\frac{1}{2}\pi$, $\frac{1}{4}\pi$, etc. (**16.4**), rather than $\pi/2$, $\pi/4$. It is not easy to bring (**16.3**)* into a single line by the use of the solidus; $r/a \geqslant e^{\frac{1}{2}\pi}$ is satisfactory, but $a/r(e^{\frac{1}{2}\pi}-\frac{1}{2}\pi)$ would be wrong, for it puts the brackets in the denominator. The alternative is $a(e^{\frac{1}{2}\pi}-\frac{1}{2}\pi)/r$, but the best solution in a single line is (16.5) in which a/r becomes ar^{-1}.

This example has been elaborated to show that an author could often write an expression in one line with no loss to the reader if he realized that it would assist the printer.

In (**16.6**)* the solidus has been used to separate the numerator and denominator of the main fraction; (16.7) shows this set correctly with the horizontal bar separating the main fraction and a solidus the subsidiary ones. There is little excuse for the unsightly solidus in (**16.8**)*. This could be reset with a horizontal fraction bar making a fraction 50 points deep, but it is an ideal instance for the use of ÷ instead of a solidus (16.9). Mathematicians frequently neglect the use of the ÷ sign where it would improve the appearance of an expression; it is recommended that it should be used in most cases where a solidus larger than the text size would otherwise have to be employed.

Example (**16.10**)* has been reset as (**16.11**) to avoid the ugly juxtaposition of the solidus and root sign. The compositor cannot be expected to change notation, but (**16.10**)* could be retained as a two-line fraction if expressed as (16.12), and this form is preferable to either (**16.10**)* or (**16.11**).

$$C = (s-1)/(s+1) \qquad (\mathbf{16.1})$$

$$C = \frac{s-1}{s+1} \qquad (16.2)$$

$$L = 2\kappa \mathrm{d}\left\{\pi/2 \frac{\mathrm{a}}{\mathrm{r}}(\mathrm{e}\,\pi/2 - {}^{\pi}/_{2})\right\};\ \frac{\mathrm{r}}{\mathrm{a}} \geqslant \mathrm{e}\,{}^{\pi}/_{2} \qquad (\mathbf{16.3})^*$$

$$L = 2\kappa d\left\{\tfrac{1}{2}\pi - \frac{a}{r}(e^{\frac{1}{2}\pi} - \tfrac{1}{2}\pi)\right\};\quad \frac{r}{a} \geqslant e^{\frac{1}{2}\pi} \qquad (\mathbf{16.4})$$

$$L = 2\kappa d\{\tfrac{1}{2}\pi - ar^{-1}(e^{\frac{1}{2}\pi} - \tfrac{1}{2}\pi)\};\quad r/a \geqslant e^{\frac{1}{2}\pi} \qquad (16.5)$$

$$A/A_T = \pi k\left(\frac{L}{T}-1\right)\Big/\sin \pi k\left(\frac{L}{T}-1\right) \qquad (\mathbf{16.6})^*$$

$$\frac{A}{A_T} = \frac{\pi k(L/T-1)}{\sin \pi k(L/T-1)} \qquad (16.7)$$

$$\frac{\delta}{\overline{A}\mathfrak{R}} = \left|\int_{\theta_m}^{\theta_0} K\,\mathrm{d}\left(\frac{\mu}{F}\right)\right/ \int_{\theta_m}^{\theta_0} K\,\mathrm{d}F \qquad (\mathbf{16.8})^*$$

$$\frac{\delta}{\bar{A}\mathfrak{R}} = \left|\int_{\theta_m}^{\theta_0} K\, d\left(\frac{\mu}{F}\right)\right| \div \int_{\theta_m}^{\theta_0} K\, dF \qquad (16.9)$$

$$t = r\Big/\sqrt{\left(\frac{1-r^2}{N-2}\right)} \qquad (16.10)^*$$

$$t = \frac{r}{\sqrt{\left(\frac{1-r^2}{N-2}\right)}} \qquad (16.11)$$

$$t = r\sqrt{\left(\frac{N-2}{1-r^2}\right)} \qquad (16.12)$$

Size of fractions

It is difficult to give an infallible guide to the compositor in choosing the size of type for fractions. There are three choices: fractions made from the body type two lines deep, fractions supplied with the text fount ($\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, etc.), and split fractions. Modern Series 7 provides the largest range of fractions as shown here:

$\frac{1}{4}\ \frac{1}{2}\ \frac{3}{4}\ \frac{1}{8}\ \frac{3}{8}\ \frac{5}{8}\ \frac{7}{8}\ \frac{1}{3}\ \frac{2}{3}\ \frac{1}{5}\ \frac{3}{5}\ \frac{1}{6}\ \frac{1}{16}\ \frac{3}{16}\ \frac{5}{16}\ \frac{7}{16}\ \frac{9}{16}\ \frac{11}{16}\ \frac{13}{16}\ \frac{15}{16}$
$\frac{1}{32}\ \frac{3}{32}\ \frac{5}{32}\ \frac{7}{32}\ \frac{9}{32}\ \frac{11}{32}\ \frac{13}{32}\ \frac{15}{32}\ \frac{17}{32}\ \frac{19}{32}\ \frac{21}{32}\ \frac{23}{32}\ \frac{25}{32}\ \frac{27}{32}\ \frac{29}{32}\ \frac{31}{32}$
$\frac{1}{10}\ \frac{2}{10}\ \frac{3}{10}\ \frac{4}{10}\ \frac{5}{10}\ \frac{6}{10}\ \frac{7}{10}\ \frac{8}{10}\ \frac{9}{10}\ \frac{10}{10}\ \frac{1}{12}\ \frac{2}{12}\ \frac{3}{12}\ \frac{4}{12}\ \frac{5}{12}\ \frac{6}{12}$
$\frac{7}{12}\ \frac{8}{12}\ \frac{9}{12}\ \frac{10}{12}\ \frac{11}{12}\ \frac{1}{64}\ \frac{3}{64}$

When there is a literal value in the denominator, e.g. $\frac{3}{x}$, the body size must be used for both numerator and denominator unless a solidus or the form $3x^{-1}$ is used. When the numerator contains a literal value but the denominator is only a number, then the form $\frac{1}{2}y$, $\frac{3}{4}x$ is preferred to $y/2$, $3x/4$. The Oxford University Press Rules suggest that the size of the fraction should conform to the symbols immediately following the fraction; this is not always a sufficient guide.†

Examples (17.1, 17.2) show the usual forms. (17.3) is an example where the fractional values inside the brackets could be split fractions, but as the factorial sign (!) is in the denominators of the fractions outside the brackets these must be two lines deep. (17.4) is an alternative form and it is a matter of opinion which is better.

$$\theta = \frac{1}{2}\int_0^x X\,dx \qquad t = -\tfrac{1}{2}\,|a|\cosh^{-1}\frac{x}{2\sqrt{|a|}} \qquad (17.1)$$

$$x^2 = \frac{3}{4}\left(\frac{y^2}{a}\right)^3 \qquad b_r = \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdots\frac{2r-1}{2r} \qquad (17.2)$$

$$y_1 = \cos\tfrac{1}{4}x^2\cdot\left\{1 + a\frac{x^2}{2!} + \left(a^2-\frac{5}{4}\right)\frac{x^4}{4!} + \left(a^3-\frac{59}{4}a\right)\frac{x^6}{6!} + \left(a^4-\frac{127}{2}a^2+\frac{585}{16}\right)\frac{x^8}{8!} + \cdots\right\} \qquad (17.3)$$

$$y_1 = \mathrm{e}^{-\frac{1}{4}ix^2}\,{}_1F_1\left(\tfrac{1}{4}-\tfrac{1}{2}ia\,;\,\tfrac{1}{2}\,;\,\tfrac{1}{2}ix^2\right) = \mathrm{e}^{-\frac{1}{4}ix^2}\left\{1 + (a+\tfrac{1}{2}i)\frac{x^2}{2!} + (a+\tfrac{1}{2}i)(a+\tfrac{5}{2}i)\frac{x^4}{4!} + \cdots\right\} \qquad (17.4)$$

Brackets

Sometimes mathematicians use one sort of bracket in a special sense and therefore give it a unique notational value. This has been done with the angular brackets ⟨⟩ which represent a special vector notation, but they can also be used generally. The open bracket, square bracket, brace, or parenthesis all have the same significance to the mathematician except in the case of determinants and matrices. The most common use of the bracket is to indicate that the terms enclosed must be treated as a single entity by the terms outside. When there are brackets within brackets the expression must be set in a readable manner using different styles of brackets to indicate which are pairs (17.4, 18.4).

A uniform series of brackets must be used throughout groups of expressions; some authorities would insist that the same sequence must be used throughout the whole work or be in the standard form of parenthesis, brace, square bracket and open bracket in that order ⟦[{()}]⟧. This uniformity may be desirable to give the compositor an invariable rule, but there are

† *The Printing of Mathematics*, Oxford University Press, 1954.

occasions when a brace can be used out of sequence to give a more easily read expression (18.4). The vertical bars | | have a special significance and are not brackets but the modulus sign $|x|$.

Brackets, parentheses and braces should be the same weight to avoid unnecessary prominence to any one pair. The stressed style of parenthesis (17.3) is considered better than the uniform Gill Sans style (**16.6**)*.

Depth of brackets

Both normal depth and full-face brackets and parentheses are used in setting mathematics. The full-face brackets can be used outside those of normal depth [([( )])], and should always be used outside the vertical bars: $(|x|)$ not $(|x|)$.

When the expression is two lines or more deep then the brackets should not be smaller than the deepest term they enclose (17.3, 17.4). Thus a fraction of the body size of two lines of 11- or 12-point would be enclosed in 24-point brackets; (**18.1**)* is an inexcusable error.

There are also occasions when there is an advantage in making the bracket larger than the depth of the terms enclosed (18.4). The deep braces help to square up the expression and make the enclosed values one visual group. Example (18.4) also shows a mixture of two sequences; the denominator has parentheses followed by square brackets; in the other part of the expression the braces follow parentheses. This inconsistency is considered justifiable by the form of the equation, for if brackets larger than the depth are to be used it is best that they should be braces which lead the eye to the centre line.

Authors should assist the printer to improve the appearance of expressions by avoiding unnecessarily deep brackets or parentheses. Example (**18.2**)* has brackets that are too large and (18.3) is the same equation reset with smaller brackets which have been made possible by the slight change in notation.

$$\frac{\mathrm{G}}{\mathrm{K}}\Big(\log\frac{\mathrm{r}}{\mathrm{b}}+\frac{1}{2}\Big)-\frac{\mathrm{y_{eo}}}{\mathrm{Y}} \qquad (\mathbf{18.1})^*$$

$$-200=\frac{211{,}000}{bd}\left(1-\frac{31{\cdot}5\frac{d}{2}}{\frac{d}{6}}\right) \qquad (\mathbf{18.2})^*$$

$$-200=\frac{211{,}000}{bd}\left(1-\frac{31{\cdot}5\times\frac{1}{2}d}{\frac{1}{6}d}\right) \qquad (18.3)$$

$$v=V\left[1-\sum_{1}^{\infty}\frac{\pi J_1^2(b\beta_s)}{[J_1^2(b\beta_s)-J_0^2(a\beta_s)]}\Big\{J_0(b\beta_s)Y_0(a\beta_s)-Y_0(b\beta_s)J_0(a\beta_s)\Big\}\exp(-\kappa\beta_s^2 t)\right] \qquad (18.4)$$

Special Notations

The following examples show some of the special notations used by mathematicians and illustrate the printer's difficulties in setting them.

The vinculum

Root sign. There are two conventional ways of setting the root sign when it includes several values:

$$\sqrt{x^2+y^2+z^2} \quad \text{alternative} \quad \surd(x^2+y^2+z^2) \qquad (19.1)$$

The bar (vinculum) is sometimes preferred for elementary mathematics, but there is always the danger of it slipping due to leads not being cut accurately or the rule riding over them. Although it is easier for the printer to use parentheses it is not quite so easy for the young student to follow the notation.

Roots other than square roots need a superior figure or letter and the root sign must be mortised unless the printer has matrices which include the number or letter with the root sign.† Authors should use the notation typified by $(x+y)^{\frac{1}{3}}$ whenever possible.

Tied values. Authors sometimes use the bar over two or more values to tie them together in the same manner as if they were enclosed in brackets. This is not a good alternative to brackets for the reasons given for avoiding the bar in roots. Example (**20.1**) shows two values tied with a bar and (20.2) is the same equation reset with brackets replacing the bar.

Continued fractions. The bar over several values is also used as one of the notations for continued fractions. Examples (20.3, 20.4, 20.5) are all mathematically identical and show three different notations

† The Mathematical Sorts List includes a list of these matrices.

for continued fractions. The bar as used in (20.5) cannot be replaced by brackets for it has a different meaning to the bar used in (**20.1**). Attention is drawn to the plus signs (20.4). The first one is in the usual position but the others are associated with the denominator and appear under the fraction bar.

$$\frac{\partial S}{\partial X} = V\frac{\partial e}{\partial T}\left\{1 - X\frac{\partial}{\partial X}(2ex - \overline{e_1 + e_2})\right\} \qquad (20.1)$$

$$\frac{\partial S}{\partial X} = V\frac{\partial e}{\partial T}\left\{1 - X\frac{\partial}{\partial X}[2ex - (e_1 + e_2)]\right\} \qquad (20.2)$$

$$\frac{24 - \sqrt{15}}{17} = 1 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{3}}}}} \qquad (20.3)$$

$$= 1 + \frac{1}{5+}\,\frac{1}{2+}\,\frac{1}{3+}\,\frac{1}{2+}\,\frac{1}{3} \qquad (20.4)$$

$$= 1, 5, \overline{2, 3} \qquad (20.5)$$

Calculus

Most of the difficulties in setting calculus occur in the integral calculus. The 24-point integral sign is suitable for most displayed formulae; a larger one is rarely necessary. This sized sign with a single-line equation allows sufficient space for the limits to be set in 6-point at the side of the sign. When the body fount is 11-point, the 6-point limits can be cast on $6\frac{1}{2}$-point. This permits easy make-up with the body type to the 24-point integral sign. These small figures define the limits within which the expression is integrated. They should appear to the right of the integral sign and should be set close to it and range with the top and bottom of the sign. The expression following the integral sign must also be to the right of the limits. Literal values in the limits should be set in italics.

Sometimes the limits are put above and below the integral sign; this is not wrong if done consistently, but it is not the more usual style.†

When there are double and triple integrals, setting the limits above and below the integral sign is a definite advantage, and gives an improved appearance to the formulae, but if used it must be the style for all integrals in the book. These two styles are shown in (21.1, 21.2). In some advanced mathematics an inferior letter is used under the integral sign.

When there is a two-line fraction next to the sign, sufficient space must be allowed to separate integral sign and fraction. The integral sign should be the depth of the deepest line in the equation, and when several integral signs appear in the same equation they should all be the same depth even if the equation makes more than one line.

In calculus the square bracket has a special significance. The values enclosed in square brackets when limits are shown at the side represent a quantity to be evaluated at those limits and the value at the lower limit to be subtracted from the value at the upper limit. As the use of square brackets (22.1) is thus a mathematical convention they should not be changed for braces or parentheses:

$$\int_a^b \frac{1}{x}\,dx = [\log x]_a^b = \log b - \log a \qquad (22.1)$$

The compositor must not confuse the partial differentiation sign (∂) with the Greek delta (δ); they have an entirely different significance.

Summation of series

With this notation the Greek sigma is used to indicate that the series following this sign is to be summed. The value of the summation variable for the first term of the series is put under the sigma and the corresponding value for the last term is put above it. Italic should be used for literal values, and the two lines should be centred above and below the sigma.

Sigmas are sometimes used without the first and last terms. The minimum size is not then so important, but the size should be consistent for a series of equations and would normally be the body size of the text. The 11-point mathematical summation sign is as large as a 13-point Greek capital sigma (90-13, matrix 549).

$$-\frac{dI(\vartheta,\phi)}{\kappa\rho\,ds} = I(\vartheta,\phi') - \frac{1}{4\pi}\int\limits_{0}^{\pi}\int\limits_{0}^{2\pi} P(\vartheta,\phi;\vartheta',\phi')\,I(\vartheta',\phi')\sin\vartheta'\,d\vartheta'\,d\phi' \qquad (21.1)$$

$$-\frac{dI(\vartheta,\phi)}{\kappa\rho\,ds} = I(\vartheta,\phi') - \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi} P(\vartheta,\phi;\vartheta',\phi')\,I(\vartheta',\phi')\sin\vartheta'\,d\vartheta'\,d\phi' \qquad (21.2)$$

† It is, however, the style used by the Oxford University Press and *Acta Mathematica*, Uppsala.

The difficulty of setting first and last terms in a size that will not overpower the sigma makes the use of sigmas smaller than 11-point full-face undesirable unless there are no first and last terms.

The Cambridge University Press prefer to use an 11-point full-face sigma in all expressions even when it is followed by two-line fractions; many authors would object to this practice and prefer an 18-point sigma. The 11-point full-face sigma requires less time to make up than the 18-point one and also takes up less space, and these are points worth considering when the manuscript contains many summation signs.

$$r_a = \sum_{i=1}^{N-1} (x_i x_{i+1})/\{(N-1)\sigma_x^2\} \qquad (23.1)^*$$

$$f(x) = \frac{4aKz}{\pi} \sum_{r=1}^{\infty} \frac{\sin\{(2m-1)(2n-1)\}\epsilon}{(2m-1)(2n-1)} \qquad (23.2)^*$$

$$f(x) = \sum_{r=1}^{n-1} \frac{1}{(n-r)!\ r!} (p_N - p_n)^{n-r}\, p_n^r \qquad (23.3)^*$$

Examples (**24.1**) and (24.2) show alternative settings of (**23.1**)*; in (24.2) the solidus has been replaced by the horizontal fraction bar and an 18-point sigma used. It is considered that for a displayed equation (24.2) is better than (**24.1**) and both are better than (**23.1**)*. In (24.3), which is the reset (**23.2**)*, the sigma has been increased in size to 18-point, and in (**24.4**), which is the reset (**23.3**)*, it has been reduced to 18-point. The first and last terms are in 6-point. Example (24.5) shows a better arrangement of (**24.4**), and the author should have presented the equation in this form.

$$r_a = \sum_{i=1}^{N-1} (x_i x_{i+1})/[(N-1)\sigma_x^2] \qquad (24.1)$$

$$r_a = \sum_{i=1}^{N-1} \frac{x_i x_{i+1}}{(N-1)\sigma_x^2} \qquad (24.2)$$

$$f(x) = \frac{4aKz}{\pi} \sum_{r=1}^{\infty} \frac{\sin[(2m-1)(2n-1)]\epsilon}{(2m-1)(2n-1)} \qquad (24.3)$$

$$f(x) = \sum_{r=1}^{n-1} \frac{1}{(n-r)!\,r!} (p_N - p_n)^{n-r} p_n^r \qquad (24.4)$$

$$f(x) = \sum_{r=1}^{n-1} \frac{(p_N - p_n)^{n-r} p_n^r}{(n-r)!\,r!} \qquad (24.5)$$

Combinations and permutations

In elementary work these are represented by C or P; these letters are capitals and are preceded by a superior letter or number and followed by an inferior letter or number.

$$^4C_2, \quad ^8P_1, \quad ^nC_r, \quad ^{n+1}P_{r-1} \qquad (25.1)$$

In advanced work combinations are written as fractions without the bar and enclosed in parentheses thus:

$$\binom{4}{2}, \quad \binom{n}{r}, \quad \binom{n}{s}^2 \geqslant \binom{n}{s-r}\binom{n}{s+r} \qquad (25.2)$$

Coordinates, arguments and parameters

In analytical geometry, reference is frequently made to the coordinates (x, y). These coordinates may carry the plus or minus signs $(-x, y)$ and primes (x', y'), (x'', y''). If the compositor knows that the prime is likely to be used he will not mistake it for a superior 1. The arguments of a function "a, b, c", etc., are spaced similarly to coordinates, as are a set of parameters "a, b, c".

The plus or minus signs do not describe the operation of adding or subtracting x and y but their condition as a positive or negative value and should be close up to the values. The coordinates should be separated by a 3-unit space after the comma separating the "x" and "y" with no comma after the "y". Figures are also used in the same manner and spacing should be the same as for the x and y. Example (26.1) shows correctly set arguments (cf. 21.1, 21.2). Note that the suffixes lambda and double lambda are directly under the primes.

$$f(x, y, \lambda) + f'_\lambda(x, y, \lambda)\,\delta\lambda + f''_{\lambda\lambda}(x, y, \lambda)\frac{(\delta\lambda)^2}{2!} + \cdots = 0 \qquad (26.1)$$

Matrices and determinants

As far as the printer is concerned these are small tables, set without rules but with parentheses, brackets or straight lines at the side. Determinants have one or two vertical lines on each side and matrices have square brackets or large parentheses; braces must never be used.

This notation is a method of writing a complicated expression in a simple manner by showing the components of the expression in lines and columns, and the way of setting them down indicates that the values are multiplied together according to definite rules.

As a general rule positive values do not take a plus sign before them but negative values take the minus sign. There are some authors, however, who prefer to use the plus sign and the printer should follow copy.

$$\begin{vmatrix} x & x & x & x \\ y & y & y & -y \\ z & z & -z & -z \\ t & -t & -t & -t \end{vmatrix} = 8xyzt \qquad (27.1)$$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \qquad (27.2)$$

$$\begin{pmatrix} \cos^2\Theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\Theta & 0 \\ 0 & 0 & 0 & \cos\Theta \end{pmatrix} \begin{pmatrix} \cos^2 i_1 & \sin^2 i_1 & -\frac{1}{2}\sin 2i_1 & 0 \\ \sin^2 i_1 & \cos^2 i_1 & \frac{1}{2}\sin 2i_1 & 0 \\ \sin 2i_1 & -\sin 2i_1 & \cos 2i_1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (27.3)$$

$$P(\vartheta, \phi\,;\vartheta', \phi') = \frac{3}{2}\begin{pmatrix} (b,b)^2 & (n,b)^2 & (b,b)(n,b) & 0 \\ (b,n)^2 & (n,n)^2 & (b,n)(n,n) & 0 \\ 2(b,b)(b,n) & 2(n,n)(n,b) & (b,b)(n,n)+(n,b)(b,n) & 0 \\ 0 & 0 & 0 & (b,b)(n,n)-(n,b)(b,n) \end{pmatrix} \qquad (27.4)$$

The minus or plus signs should be close up to their associated values. There should be a minimum space of 18 units between widest values in adjacent columns and of 9 units between outside rules and the widest values in the first and last columns. Medium-face rules should be used for the vertical lines. Where there are many suffixes and powers in the terms of the matrix or determinant there should be a 3-point lead between the lines (27.3).

Trigonometrical functions should range under each other, with the numerical values centred under them (27.3). When the minus sign only is used it should stand out beyond the ranged columns of terms and the sign should be close up. When several terms in a determinant are missing, the ellipsis is indicated by three-dot leaders ranged as shown in (27.2).

A style is sometimes adopted in which commas are put after each value; this is not recommended. There should be no punctuation after each group, although it may be required between grouped parameters, as example (27.4) indicates. A 3-unit space is necessary after the comma in each case.

Vector analysis and tensor calculus

Bold type is used for literal values of vector quantities and ordinary-weight type for scalar values. Some authors will insist on using German characters, but this is not a rigid convention. The Royal Society recommend bold sans ($\mathsf{\mathbf{A}} \wedge \mathsf{\mathbf{B}}$) for vectors but the related bold to the text is often used ($\mathbf{A} \wedge \mathbf{B}$). In more elementary mathematics vector quantities are shown as $\overrightarrow{AB}$, $\overrightarrow{BA}$. This notation is simple for the author when writing his manuscript but difficult for the printer to set. Example (28.1) shows a vector equation using Imprint Bold Series 410 for vectors.

$$\frac{\partial}{\partial t}(n_q \mathbf{u}_q^{(i)}) = n_q \frac{d\mathbf{u}_q^{(i)}}{dt} - \sum_{j=1}^{q} \frac{\partial}{\partial \mathbf{x}^{(j)}} \cdot (n_q \mathbf{u}_q^{(j)} \mathbf{u}_q^{(i)}) \qquad (28.1)$$

TIMES SERIES
4-LINE MATHEMATICAL SYSTEM

WITH a view to reducing the cost of composition and make-up of mathematical setting, The Monotype Corporation Limited has been experimenting for some time with what is known in the U.S.A. as the Patton Method.

The basis of this system is that the formulae are made up of four lines of 6-point, and as we have selected Times New Roman Series 327, the size of the main characters has been made 10-point instead of the more usual 11-point of Modern Series 7.

Throughout the lengthy period of investigation and trial it has been clearly borne in mind that learned bodies in this country have for many years been accustomed to a very high typographical standard. The new system if it was to be successful had to follow as nearly as possible what was being already achieved by orthodox methods, and the four-line sample on the opposite page shows a typical mathematical setting. Mathematicians will be quick to note that certain liberties have been taken with the formulae, but this is intentional and was merely done to make as representative a setting as possible.

The text is set from a separate layout although sufficient roman lower-case will be in the formulae layout to enable the keyboard operator to set lines at one operation where both text and formulae occur.

The overhanging part of the 10-point formulae characters, when cast on 6-point, has to be supported by a shoulder-high space of identical unit width, and an attachment has been devised which enables the keyboard operator to obtain these shoulder-high spaces without any calculations.

It will be seen that in making the formulae to cast on 6-point the number of superior and inferior matrices is greatly reduced in that superiors and inferiors both of first and second order are cast from the same matrices, the characters becoming superior or inferior according to the line in which they appear, i.e. a matrix used as a first-order superior can also be used as a first-order inferior; similarly, a second-order superior can be used as superior to first-order inferior and second-order inferior also functions as inferior to first-order superior.

In order to maintain the accepted face sizes and relative body positions insisted on in this country we have had to dispense with the rule as an integral part of the matrix, which is the method used in the U.S.A. Instead 2-point strip rule is used so that the depth of the equation becomes 26 points and the large characters have to be cast on this size.

As the number of useful characters in the matrix case is increased through the interchangeability of superior and inferior matrices, it follows that a considerable saving is effected at case as fewer special signs have to be inserted by hand; in fact, with the absence of "half whiting" there should be little left for the compositor to insert except the large signs and the strip rule.

The example illustrated has been time-studied by a well-known firm of specialists in mathematical setting and it was estimated that the four-line method would show a saving of 26 per cent in the time taken to keyboard, cast and make up. When the specimen was time-checked and compared with the orthodox method employed by this firm the saving was even greater than had been estimated.

In view of the great variety and different stages of complexity in mathematical setting, it is confidently felt that a fair estimate of the benefit to be obtained by the four-line system will be in the neighbourhood of 20 per cent, although this figure will be improved for complicated settings and reduced a little for simple work.

Inquiries for further particulars are invited and should be addressed to our Service "A" Department, Salfords. It should be understood, however, that so far only sufficient matrices have been made in order to produce this specimen, and in view of the large amount of work still to be undertaken before actual manufacture can start, it is not anticipated that the full range of equipment will be available before about the middle of 1958.

NOTE: Since this Mathematical Number first appeared, continuing demand has necessitated two reprintings. To this present one we append, as a loose inset, a copy of the latest INFORMATION SHEET (No. 156, March, 1959), giving further and fuller details of the 4-Line System – which has now been adopted by many well-known houses. ED.

The above solution is appropriate for determining $g(t)$ when $g_0, g_1, \ldots, g_{m-1}$ are given, but in the application that is made here to Fourier analysis it is the function $g(t)$ that is given and the transform that is required. The typical functions occurring in $g(t)$ are $t^p e^{at} \cos bt$ and $t^p e^{at} \sin bt$, and the transforms for these are easily obtained from that of the simple function e^{kt}. Thus

(i) if $g(t) = e^{kt}$, then $F(z, t_0) = \dfrac{e^{kt_0}}{z-k}$;

(ii) if $g(t) = t^p e^{kt}$, differentiation with respect to k of the above result shows that

$$F(z, t_0) = \left\{\frac{t_0^p}{z-k} + \frac{p t_0^{p-1}}{(z-k)^2} + \frac{p(p-1)t_0^{p-2}}{(z-k)^3} + \ldots + \frac{p!}{(z-k)^{p+1}}\right\} e^{kt_0};$$

(iii) the transforms of $t^p e^{at} \cos bt$ and $t^p e^{at} \sin bt$ are the *apparent* real and imaginary parts of

$$\left\{\frac{t_0^p}{z-a-ib} + \frac{p t_0^{p-1}}{(z-a-ib)^2} + \ldots + \frac{p!}{(z-a-ib)^{p+1}}\right\} e^{t_0(a+ib)},$$

(*i.e.*, ignoring for the moment the occurrence of i in z). For example:

(i) $g(t) = P(t)$, a polynomial of degree p,

$$F(z, t_0) = \frac{P(t_0)}{z} + \frac{P'(t_0)}{z^2} + \ldots + \frac{P^{(p)}(t_0)}{z^{p+1}};$$

(ii) $g(t) = t^2 \sin \tfrac{1}{2}t$,

$$F(z, t_0) = \mathrm{I}' \left\{\frac{t_0^2}{z-\frac{1}{2}i} + \frac{2t_0}{(z-\frac{1}{2}i)^2} + \frac{2}{(z-\frac{1}{2}i)^3}\right\} e^{\frac{1}{2}it_0},$$

(the accent denoting that the apparent imaginary part is taken)

$$= \mathrm{I}' \left\{\frac{t_0^2(z+\frac{1}{2}i)}{z^2+\frac{1}{4}} + \frac{2t_0(z^2-\frac{1}{4}+zi)}{(z^2+\frac{1}{4})^2} + \frac{2\{z^3-\frac{3}{4}z+i(\frac{2}{3}z^2-\frac{1}{8})\}}{(z^2+\frac{1}{4})^3}\right\} e^{\frac{1}{2}it_0}.$$

Thus $$F(z, 0) = \frac{16(12z^2-1)}{(4z^2+1)^3},$$

and $$F(z, \pi) = \frac{4\pi^2 z}{4z^2+1} + \frac{8\pi(4z^2-1)}{(4z^2+1)^2} + \frac{32z(4z^2-3)}{(4z^2+1)^3}.$$

It may be noted, however, that

$$\frac{1}{2\pi i}\int_{\Gamma_N} \frac{dz}{z(e^{2\pi z}-1)} = -\tfrac{1}{2} \quad \text{and} \quad \frac{1}{2\pi i}\int_{\Gamma_N} \frac{e^{2\pi z}\, dz}{z(e^{2\pi z}-1)} = +\tfrac{1}{2}$$

for all $N > 0$.

Putting $t = \theta$ in the residue equation we have

$$\tfrac{1}{2}\alpha_0 + \sum_{1}^{N} (\alpha_n \cos nt + \beta_n \sin nt) = \sum_{0}^{m-1} I_r - I_m,$$

where $$I_0 = \frac{1}{2\pi i}\int_{\Gamma_N} \frac{g(t_1')\, e^{\theta z}\, dz}{z(e^{2\pi z}-1)}, \quad I_r = \frac{1}{2\pi i}\int_{\Gamma_N} \frac{\{g(t_{r+1}')-g(t_r')\}\, e^{z(\theta-t_r)}\, dz}{z(e^{2\pi z}-1)},$$

$$I_m = \frac{1}{2\pi i}\int_{\Gamma_N} \frac{g(t_{m_p}')\, dz}{z(e^{2\pi z^h}-1)}.$$

A TIMES SERIES 4-LINE SPECIMEN

Names or descriptions of Mathematical Signs

AN attempt has been made to name all the mathematical signs which have been cut by The Monotype Corporation Limited, but this has not been possible. Signs have been given different meanings by different authors, new signs are continually coming into use and others have only an ephemeral meaning. With a few exceptions this list excludes Greek, roman or italic letters which are conventionally used for symbols representing mathematical or physical values. A comprehensive list of these is given in the British Standards Institution publications B.S. 1991, and in The Royal Society publication *Symbols, signs and abbreviations recommended for British scientific publications.*

It is hoped that the following list of mathematical signs with their description or name will be of considerable assistance to printers in providing a verbal description of a sign and also of assistance to authors; they will now be able to see the range of signs available and thus avoid incurring the delay involved in producing new matrices unless it is absolutely necessary for their notation.

In order to avoid misunderstandings that may arise from published extracts, a written request should be made to The Monotype Corporation Limited for permission to reproduce any part of the list.

Where signs have more than one use, the several meanings are given, but their order does not indicate that the first meaning is in more general use than any of the others. As far as is known, obsolete meanings have been avoided. The list is not in matrix-number sequence, but similar signs are grouped together. Many signs are cut in several different styles in addition to those shown (e.g. + **+**). In such cases the one shown is that which is most commonly used. Further information on sizes and alternative designs is given in the Mathematical Sorts List which is published separately and can be obtained on application to The Monotype Corporation Limited. A postcard is enclosed in this issue of the RECORDER for that purpose.

Symbol and matrix number	Description	Symbol and matrix number	Description
s4535 ′	Prime	s96 {	Brace
s64 ″	Double prime	s95 }	Brace
s2945 ‴	Triple prime	s318 ⟨	Angular bracket
s6132 ⁗	Quadruple prime	s317 ⟩	Angular bracket
s4477† ‚	Inferior prime	s3185 ⟦	Open bracket
s4478† „	Inferior double prime	s3186 ⟧	Open bracket
s4479† ‚‚‚	Inferior triple prime	s3133 *⟦*	Italic open bracket
s4475 ‵	Reversed prime	s3134 *⟧*	Italic open bracket
s4536 °	Degree	s1955 ≬	Between
s229 ∵	Because or since	s234 ∞	Infinity
s3696† ∴	Therefore	s3573 $\bar{\infty}$	
s5012 :	Sign of proportion	s233 ∝	Varies as, proportional to
s230 ∷	Sign of proportion	s216 ·	Decimal point
s231 ∻	Geometric proportion	.	Scalar product (full point)
s5027 ⋮		!	Factorial sign
s2782 **[**	Full-face bracket	s113 $\vert$	(1) Modulus (used thus: $\vert x \vert$)
s2783 **]**	Full-face bracket		(2) Joint denial (math. logic, thus: $p \vert q$)
s2784 **(**	Full-face parenthesis		(3) Divides (number theory, thus: $3 \vert 6$)
s2785 **)**	Full-face parenthesis	s161 $\Vert$	Parallel to

Symbol and matrix number	Description
S120 /	Divided by (solidus)
S243 $\sqrt{}$	Square root
S245 $\sqrt[3]{}$	Cube root
S246 $\sqrt[4]{}$	Fourth root
S7492 $\sqrt[n]{}$	*n*th root
S3460† +	Plus
S3461† −	Minus
S7111 ⤼	
S7110 ⤽	
S3462† ±	Plus or minus
S2454 ∓	Minus or plus
S2477 $\underline{\underline{\pm}}$	
S8431 $\overline{\overline{\mp}}$	
S6497 ⊕	(1) Sign of composition
	(2) Direct sum
S6498 ⊖	
S6420 ⊗	Plethysm operator (group theory)
S8137† ∔	Direct sum (group theory)
S8831 ⨢	
S5770† $+_2$	Nim-addition
S9908‡ ⨣	
S3463† ×	Multiply
S3464† ÷	Divide
S3465† =	(1) Equal to
	(2) Logical identity
S3561† ≠	Is not equal to
S8682 =⃥	Is not equal to
S3535† ǂ	Logical diversity
S6772 ⫧	
S6762 ╪	
S7514 ≈	(1) Approximately equal to
	(2) Asymptotic to
	(3) Equal to in the mean
S6654† ≅	(1) Similar to
	(2) Geometrically equivalent, congruent to
	(3) Equal or nearly equal to
S7028 ≎	Geometrically equivalent to
S10090 ≙	Approximately equal to or equal to
S5808 ≍	Equivalent to
S8449 ⪥	
S9902† ≊	Approximately equal to or equal to
S5515† ≑	Approximately equal to
S5864† ≒	Approximately equal to

Symbol and matrix number	Description
S6773 ≏	Approximately equal to
S5121† ≃	(1) Approximately equal to
	(2) Asymptotic to
S8103 ∼⃒	Not asymptotic to
S5119† ∼	(1) Difference between
	(2) Is equivalent to
	(3) Asymptotic to
	(4) Similar to
	(5) Of the order of
	(6) The complement of
	(7) Is not, Negation sign (math. logic)
S5120† ≁	(1) Is not equivalent to
	(2) Is not asymptotic to
	(3) Is not similar to
	(4) Is not the complement of
S9569† ⩪	
S5771 ⩫	Approximately equivalent to
S9567† ⪆	Equivalent to or greater than
S9568† ⪅	Equivalent to or less than
S9133 ≳	Greater than or equivalent to
S9134 ≲	Less than or equivalent to
S9829‡ ⩾	Greater than or equivalent to
S9828‡ ⩽	Less than or equivalent to
S9570† +̃	Positive difference or sum
S7154 ±̰	Sum or positive difference
S6894 ≿	
S5094 ≻	(1) Has a higher rank or order
	(2) Contains
S7478‡ ⊀	(1) Has not a lower rank or order than
	(2) Is not contained in
S8451 ≼	Is contained in, or equal to
S232 ⪕	Smaller than
S3466† <	Less than
S3467† >	Greater than
S4862† ≮	Not less than
S4863† ≯	Not greater than
S4596† ≪	Much less than
S4597† ≫	Much greater than
S7656 ≫̸	Not much greater than
S2905 ⋛	Greater than or less than
S297 ⋚	Less than or greater than
S3523† ≦	Less than or equal to
S3527† ⩽	Less than or equal to

Symbol and matrix number	Description
s3524† ≧	Greater than or equal to
s3528† ⩾	Greater than or equal to
s8047 ⪋	Less than, equal to, or greater than
s8048 ⪌	Greater than, equal to, or less than
s2807 ⋛	Greater than, equal to, or less than
s319 ⪑	Less than, greater than, or equal to
s320 ⪒	Greater than, less than, or equal to
s8102 ≱	Not greater than nor equal to
s9577 ≛	
s6680 ≐	(1) Approaches the limit
	(2) Approaches in value to
s6759‡ ⨪	
s6032† ⊂	(1) Is implied by
	(2) Contained as proper sub-class within
s6033† ⊃	(1) Implies
	(2) Contains as proper sub-class
s5769† ⊆	(1) Contained as sub-class within
	(2) Is identical to
s6031† ⊇	(1) Contains as sub-class
	(2) Is identical to
s5503 ⫔	Contains or is contained in
s3397 ⫓	
s5929 ⫗	
s6938 ⊄	Is not contained in
s6939 ⊅	Does not contain
s6958 ⊈	(1) Is not contained as sub-class within
	(2) Is not identical to
s6959 ⊉	(1) Does not contain as sub-class
	(2) Is not identical to
s9162 ⪽	Is included in, as sub-relation (math. logic)
s9161 ⪾	Includes as sub-relation (math. logic)
s8153 ∩	Product or intersection, or meet of two classes (math. logic) or sets (algebra). Colloquially called "cap"
s8154 ∪	Sum or union or join of two classes (math. logic) or sets (algebra). Colloquially called "cup"
s4847 ⋐	
s5261 ⊢	What follows is true, Assertion sign (math. logic)
s8344‡ ⋗	
s6678 ⪖	
s6679 ⪕	

Symbol and matrix number	Description
s9343 ⋖	
s9344 ⋗	
E945 ∃	There exists
s5546 ∈	Is an element of
s8409 ∉	Is not an element of
s5547 ⋹	Is not an element of
s3534† ≡	(1) Is congruent to
	(2) Definitional identity (math. logic)
	(3) Is identical with
	(4) Is equivalent to (math. logic)
s10477† ≢	(1) Is not congruent to
	(2) Is not identical with
s5453† ∤	Does not divide
s8453 ⫽	
s6693 ⧣	Is homothetically congruent to
s9903† ⋕	Equal and parallel
s6694 ⧥	Congruent and parallel
s2473 →	(1) Approaches or tends to the limit
	(2) Implies (math. logic)
	(3) Referents of a relation (math. logic, used thus: $\overrightarrow{R}$)
s9571 ⇸	Does not tend to
s2472 ←	Relata of a relation (used thus: $\overleftarrow{R}$)
s7352 ↑	Tends up to the limit
s7353 ↓	Tends down to the limit
s5575 ↕	
s5016 ⇔	Implies and is implied by
s9575 ⇒	Converges to
s5017 ⇐	Is implied by
s5772 ↔	(1) Mutually implies
	(2) One-to-one correspondence with
	(3) Corresponds reciprocally
	(4) Asymptotically equivalent to
s9573 ⇹	
s3454 ↻	Clockwise
s3455 ↺	Anti-clockwise
s585 ∧	(1) Vector product
	(2) Product of two sets (math. logic)
	(3) Symmetric difference of two sets (math. logic)
s9578 ≙	Estimates or is estimated by
s9579 ⩮	
s2845 ⊻	Equiangular (geometry)

Symbol and matrix number	Description	Symbol and matrix number	Description
s7527 ⊼	Is projective with or projective correspondence	s223 □	(1) D'Alembertian operator (2) Mean operator (finite differences)
s1163 ⊼	Perspective correspondence	s8450 ⅄	
		s8452 ⚹	
s2559 ∫	Integral sign	Gamma Γ	Gamma function
		D36 ∂	Partial differentiation sign
s3072 ∮	Contour integral	Delta Δ	Increment or forward finite-difference operator
s9975 ∯	Double contour integral	Δ48 ∇	Nabla or del or backward finite-difference operator
s9781 ∳	Contour integral (anti-clockwise)	θ233 ϑ	Theta function
s9795 ∲	Contour integral (clockwise)	Π197 ∏	Product sign
		Σ423 ∑	Summation sign
s5331 ⨍		F456 Ϝ	Digamma function
		A244 ℵ	Aleph. The number of finite integers is $\aleph_0$ and transfinite cardinal numbers $\aleph_{1,2,3\ldots}$
s5332 ⨎		C275 ℘	Weierstrass elliptic function
s5333 ⨗		&	Conjunction of statements (math. logic)
		∨	(1) Disjunction of statements (math. logic) (2) Sum of two sets (math. logic)
s5334 ⨕		O	Of order (used thus: $O(x)$)
s6575 ⨐		o	Of lower order than (used thus: $o(x)$)
s7132 ⨒		f	Function of (used thus: $f(x)$)

† and ‡ indicate that the signs match.

Mathematical Abbreviations

Several mathematical functions and operations are not indicated by signs but by abbreviations which are set in roman. Capitals are not used for the initial letter except where shown in the following list. A full point should not be used after any mathematical abbreviation, for it is a multiplication sign. Although not standard practice, some editors use roman for the *d* in *dy*/*dx*, it being considered that all operators and constants should be roman to distinguish them from values. By the same reasoning "i" or "j" ($\sqrt{-1}$) and the exponential "e" are set in roman.

Trigonometric functions

sine	sin
cosine	cos
tangent	tan
cosecant	cosec
secant	sec
cotangent	cot
haversine	hav
versine	ver

Hyperbolic functions

sine	sinh
cosine	cosh
tangent	tanh
cosecant	cosech
secant	sech
cotangent	coth

Miscellaneous

Airy integral	Ai, Bi
amplitude	am
antilogarithm	antilog
argument	arg
Bessel function	Kh
cosine integral	Ci or ci
critical	crit
divergence	div
elliptic functions	sn, cn, dn
exponential	exp
exponential integral	Ei
gradient	grad
imaginary part	Im or $\mathscr{I}$
limit	lim
logarithm	log
logarithmic integral	Li
maximum	max
minimum	min
modulus	mod
natural logarithm	ln
optimum	opt
phase	ph
real part	Re or $\mathscr{R}$
sign	sgn
sine integral	Si or si
ultimate	ult

The words "arc" and "curl" should also be set in roman although they are not abbreviations.

Glossary

ANTILOGARITHM. See LOGARITHM.

ARGUMENT. See FUNCTION.

ASYMPTOTIC. Two quantities are said to be asymptotic to each other if their ratio tends to unity. The sign $\sim$ is used to denote asymptotic equality.

Ex. x^2+x is asymptotic to x^2 for large values of x. This is written symbolically as $x^2+x \sim x^2$ as $x \to \infty$.

COMBINATION. The total number of different ways in which a prescribed number of objects can be selected from a given set is called a combination.

Ex. The number of different pairs of cards which can be selected from a suit of 13 is a combination and is denoted by

$$^{13}C_2 \quad \text{or} \quad \binom{13}{2}$$

CONSTANT. A mathematical or physical quantity which does not change its value, e and π are constants.

CONTINUED FRACTION. This is a fraction which has one or more fractions in its denominator.

Ex. $\cfrac{1}{2+\cfrac{3}{5+\cfrac{1}{4}}}$ is a continued fraction. It is conventionally written

$$\frac{1}{2+}\,\frac{3}{5+}\,\frac{1}{4}$$

COORDINATES. The position of a point on a piece of graph paper can be fixed by giving its distances from one vertical and one horizontal edge. These two distances are called rectangular coordinates. The edges used are termed the *axes* and their point of intersection is the *origin*.

Similarly, the position of a point at the end of a hand on a clock face could be fixed by giving the length of the hand and the angle it makes with the twelve o'clock position; these measurements are polar coordinates.

DENOMINATOR. The lower half of a fraction.

Ex. The figure 3 is the denominator of $\frac{2}{3}$; $x+a$ is the denominator of $\dfrac{x}{x+a}$.

DERIVATIVE OR DIFFERENTIAL COEFFICIENT. The derivative or differential coefficient of a quantity with respect to time is the rate of change of that quantity at any instant. Thus the speed (v) of a cyclist is the derivative with respect to time of his distance (s) from the starting-point. This is written either

$$v = \frac{ds}{dt} \quad \text{or} \quad v = \dot{s}.$$

Similarly his acceleration (a) is the derivative of his speed with respect to time; it is also the *second* derivative of the distance, and this is written

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \quad \text{or} \quad a = \dot{v} = \ddot{s}.$$

Derivatives also exist with respect to quantities other than time. The equations just given are examples of *differential equations.*

DETERMINANT. This is a concise notation used in writing down the solution of several simultaneous equations. It consists of a square array of numbers or letters enclosed by vertical rules. The numbers or letters are called the *elements*, and the value of the determinant is obtained by multiplying and adding them together according to certain mathematical rules.

e OR EPSILON. This is the base of natural logarithms (see LOGARITHM). The value of e is 2·71828 approximately. The antilogarithm of a variable x to the base e is called the *exponential function* and is denoted by e^x. It has the property that its derivative (which see) equals e^x itself.

EQUATION. This is a relation between various quantities, usually expressing unknown values in terms of known values.

EXPONENT, POWER OR INDEX. The product of a number x multiplied by itself is written x^2, and 2 is said to be the exponent or power of x in this expression. Similarly x^n means the product $x \times x \times x \ldots n$ times, and the exponent of x^n is n.

EXPRESSION. This is almost anything a mathematician writes in a mathematical way.

FACTORIAL. The factorial of a whole number is the product of that number and all the whole numbers which are less than it.

Ex. Factorial $6 = 6\times5\times4\times3\times2\times1 = 720$ and it is written 6 ! or sometimes $\underline{|6}$.

FUNCTION. One variable y is said to be a function of another variable x if an alteration of the value of x alters the value of y. Conversely x is said to be the *argument* of y. Functions can have more than one argument.

INDEX. See EXPONENT.

INTEGRAL. This is the inverse of the derivative (which see). The distance s covered by the cyclist is said to be the integral of v with respect to t and is written

$$s = \int v\,dt.$$

Inferior and superior numbers or letters may be attached to the integral sign; they give the starting and final values of t. Thus

$$\int_1^2 v\,dt$$

means the distance covered by the cyclist from time $t = 1$ to time $t = 2$.

LIMIT. A function y may approach a certain value very closely, without actually ever attaining the value, as its argument x tends to a special value. This value is then said to be the limit of y for that value of x.

Ex. When $x = 1$ the function $y = \frac{x^2 - 1}{x - 1}$ becomes $\frac{0}{0}$, which is meaningless. But for values of x close to 1, y is very close to 2 and so 2 is the limit of y as x tends to 1.

LOGARITHM. The logarithm x of a number N is the power to which the *base* must be raised in order to equal N. With the base 10 this relation is written

$$10^x = N \quad \text{or} \quad x = \log_{10} N.$$

Giving the value 3 to x we get

$$10^3 = 1000 \quad \text{or} \quad 3 = \log_{10} 1000.$$

Conversely, N is called the *antilogarithm* of x, and is written $N = \text{antilog}_{10}\, x$ or $1000 = \text{antilog}_{10}\, 3$.

Other bases than 10 are used; for example natural logarithms are those to the base e, and the relations for them are written

$$e^x = N, \text{ and } x = \log_e N \quad \text{or} \quad x = \ln N.$$

MATRIX. Written in full a matrix consists of a rectangular array of numbers bordered by either large parentheses or preferably large square brackets. The advantage of using matrix notation is that a single letter can be used to denote a whole array. (Compare DETERMINANT.)

NOTATION. The way in which an expression is written is called the notation. Sometimes there are several alternative notations, and if a printer is acquainted with them he may be able to persuade the author to adopt the easiest form of setting.

NUMERATOR. This is the top half of a fraction.

Ex. The figure 2 is the numerator of $\frac{2}{3}$.

OPERATOR. This is a symbol (which see) that has no value on its own but indicates that a mathematical operation is to be performed with the related symbols.

PARAMETER. A function y may depend on several arguments a, b, c and x. In a given problem only the variation of y with respect to x may be of interest and a, b and c may be treated as constants. They are then said to be parameters.

PARTIAL DERIVATIVE. A function of two or more variables has a derivative with respect to each of them. These derivatives are called partial derivatives and denoted by $\partial y/\partial x$ to distinguish them from the ordinary derivative dy/dx. (See also DERIVATIVE.)

POWER. See EXPONENT.

SERIES. A sequence of numbers or letters connected by the sign of addition or subtraction is called a series. The individual numbers or letters are called the *terms* of the series, and the result of carrying out all the addition and subtraction is called the *sum* of the series.

Ex. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}$ is a series; its sum is $\frac{63}{32}$.

When the law of formation of successive terms is obvious, only the first few terms are written down, the presence of the succeeding terms being indicated by inserting three dots. If the number of terms in a series is unlimited, that is if every term is followed by other terms, then the series is said to be an *infinite series*.

Ex. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$ is an infinite series whose sum is 2.

Another notation for series is the use of the Greek $\sum$. The last example would be written in this notation.

$$\sum_{n=0}^{\infty} \frac{1}{2^n} = 2.$$

The symbols above and below the $\sum$ in this example mean that n takes the values 0, 1, 2, 3, ... up to infinity.

SIGN. A mathematician's ideograph not having the form of a conventional letter or figure and expressing a mathematical relation, condition, operation or process.

SIGNIFICANT FIGURES. When a numerical approximation is given the number of digits present is called the number of significant figures, with the reservation that if the approximation is less than unity then zeros occurring between the decimal point and the first non-zero digit are not to be counted.

Ex. 2·71828 is an approximation to e having six significant figures; 0·001030 has four significant figures.

SUFFIXES. These are inferiors attached to an ordinary symbol. By using them the number of symbols available is increased. Thus x_1, x_2, x_3 may represent three distinct unknowns or variables.

Other meanings may be ascribed to suffixes, for example if they are used in conjunction with a prime. Thus y'_x is another notation for $\partial y/\partial x$.

Complicated suffixes should be avoided by authors of printed works, if necessary by the introduction of additional symbols.

SYMBOL. Any letter or sign used in a mathematical expression. In languages using the roman alphabet the letters used by mathematicians are usually selected from the italic, roman, script, Greek and Fraktur alphabets with the addition of the Hebrew aleph ($\aleph$). Letters and signs may be modified by the addition of mathematical accents.

TRIGONOMETRIC FUNCTIONS. The names of the principal trigonometric functions are listed on p. 29. They can be regarded as ratios of sides of right-angled triangles, but they have applications in many branches of mathematics.

VALUE. When a letter is used to denote a known or unknown quantity this is said to be its *literal value*. When its quantity is expressed as a number this is said to be its *numerical value*.

VECTOR. This is a physical quantity which has direction as well as magnitude. An example of a vector is the velocity of a boat travelling north-east at 10 knots.

In contrast a *scalar* is a physical quantity which has a magnitude but no direction. The mass of an object is, for example, a scalar quantity. The term 'vector' is also used to denote a single-column or single-row matrix.

Alignment and set width of Bold, Greek, and Fraktur faces compared with faces normally used for mathematics

Type face	5½-point	6-point	7-point	8-point	9-point	10-point	11-point	12-point	13-point
Modern Series 1		0·1210 6¾	0·1208 7	0·1237 8	0·1260 8½	0·1290 9¼	0·1310 9¾	0·1360 11¼	
Old Style Series 2		0·1200 6½	0·1230 7¾	0·1230 8½	0·1250 9¼	0·1295 9¾	0·1315 10½	0·1360 12	
Modern Series 7		0·1235 7	0·1210 7¾	0·1235 8½	0·1260 9¼	0·1290 9¾	0·1310 10½	0·1360 12	
Imprint Series 101		0·1237 7	0·1257 7¾	0·1285 8½	0·1312 9¼	0·1300 9½	0·1325 10¼	0·1375 11¾	
Baskerville Series 169		0·1220 6¾		0·1250 8¼	0·1266 9	0·1294 10	0·1320 10¾	0·1360 12	
Times New Roman Series 327	0·1239 6½	0·1230 6¾	0·1268 7¾	0·1298 8¼	0·1325 9	0·1338 9¾	0·1368 10½	0·1420 12	
Modern Bold Series 570		0·1235 7	0·1210 7¾	0·1235 8½	0·1260 9¼	0·1290 9¾	0·1310 10½	0·1360 12	
Old Style Bold Series 53	0·1202 6¾	0·1202 6¾	0·1250 7¾	0·1235 8½	0·1257 9¼	0·1291 9½	0·1315 10½	0·1364 12	
Old Style Bold Series 544		0·1200 6½	0·1230 7¾	0·1230 8½	0·1250 9¼	0·1295 9¾	0·1315 10½	0·1360 12	
Imprint Bold Series 410		0·1237 7	0·1257 7¾	0·1285 8½	0·1312 9¼	0·1300 9½	0·1325 10¼	0·1375 11¾	
Upright Greek Series 90		0·1235 7½		0·1250 8	0·1270 9	0·1300 9¾	0·1320 10½		0·1390 12½
Inclined Greek Series 91		0·1198 6¾		0·1245 8¼	0·1270 9		0·1320 10½		0·1390 12½
Greek Upright Display Series 92		0·1208 6¾			0·1270 9		0·1320 10½		0·1390 12½
Greek Series 472 (to work with Modern Series 7)				0·1235† 8½	0·1260† 9¼	0·1290 9¾	0·1310† 10½		
Greek Series 473 (to work with Baskerville)‡								0·1360 12	
Porson Greek Series 106	0·1195 6½			0·1250 8		0·1294 9¾	0·1315 10½	0·1356 12	
Times Greek Upright Series 565		0·1230 6¾		0·1298 8¼		0·1338 9¾		0·1420 12	
Times Greek Inclined Series 566		0·1230 6¾		0·1298 8¼		0·1338 9¾		0·1420 12	
Times Greek Bold Upright Series 567		0·1239 6¾		0·1298 8¼		0·1338 9¾		0·1420 12	
Wittenberger Fraktur Series 28			0·1255 7		0·1355 8½	0·1383 9¾	0·1418 10½		0·1473 12
Halbfette Wittenberger Fraktur Series 29			0·1268 7		0·1350 8½	0·1400 9¾	0·1427 10½		0·1490 12

† Lower-case only.

‡ Series 473 comprises the lower-case of 472 and those Greek capitals which differ from the roman and is cast to Baskerville alignment.

Note: see page 11 for comment on alignment and set.

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